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THE DETERMINATION OF ABERRATIONS, AS EXPRESSED IN GEOMETRICAL OPTICS, FROM THE INDICATIONS OF THE HILGER INTERFEROMETER*

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ABSTRACT. The paper describes and illustrates methods of deriving from the indications of the Hilger lens-testing interferometer quantitative expressions for the aberrational defects of a lens system, in terms of the departure from ideally perfect focal ray-union, as measured longitudinally at the focal plane, and as obtained in the course of computation of an objective.

ON reference to the original descriptions† of the Hilger Interferometer, and of its function, it will be seen that the aberrations of an objective under test are exhibited in a form which, however more "natural," is, to those without direct acquaintance with the working principles of the instrument, perhaps not more self-illuminative than that which is in current use and which has established itself upon the geometrical treatment of the theory of optical instruments. It is the purpose of the present communication to outline a routine method of determining the aberrations as expressed in the latter more commonly appreciated form, from measurements upon the instrument.

The testing of an optical system by the means in question depends upon the ocular demonstration of the relative retardation Δ over the cross-section of a parallel beam of light after passage through the system under test. This is brought

* The following investigation was undertaken as an extra-mural research on behalf of the British Scientific Instrument Research Association, and is published at the instance of Messrs Adam Hilger, Ltd.

† F. Twyman, *Phil. Mag.* 35 (1918) 49; *Trans. Opt. Soc.* 22 (1920-21) 174.

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about by an interferometric comparison (a) by securing conditions suitable to the production of interference between a standard beam and the beam suffering passage through the system, and (b) by establishing in the course of the interfering beams—to within practicable limits—an optical one to one correspondence between points in the cross-section of the emergent beam and points upon the recording sensitive surface, be it the human retina or the photographic plate. Interference fringes are then observed, which form what is fittingly termed a "contour map"—i.e. of the distribution of relative retardation over the cross-section of the emergent beam: in the usually adopted arrangement* the latter is caused to re-traverse the optical system and so to be twice affected by aberrational residuals in the system.

Thus the aberrations are observed, and for laboratory or industrial purposes it is very desirable that a suitable means of recording them be available. Many alternative schemes suggest themselves, but to provide for the impulse to compare the results of a test with those of calculation it is desirable† that the aberrations be reduced to linear values at the focal plane, precisely in the form resulting from numerical computation, and thus be recordable graphically in the usual way. That this is possible and, indeed, in several ways is obvious. Thus, measurements upon an interferogram may be made to yield the desired data; or otherwise, by mechanical adjustments during the course of a test, the actual quantities required, or others related to them, may be measured. The latter is the most generally applicable procedure and also the one placing fewer demands necessitating expenditure of time and material.

A small controlled adjustment may be caused to introduce, for any limited region of the emergent wave-front under contemplation, an effect equal but opposite to that presented by the lens under test, so that the relative retardation is stationary at the point and in the orientation in question. The wave-normal at the region of the firstly emergent wave-surface concerned in the adjustment is then evidently directed toward the centre of curvature of the spherical reference mirror. Various adjustments of this kind are possible: e.g. the reference mirror, intercepting the path of the converging beam proceeding from the lens under test, may receive small known adjustments, which will measure directly the quantities required—or

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Fig. 1. Obliquity 0° : normal figure.

Fig. 1 a. Obliquity 0° : setting for measurement of spherical aberration by adjustment to deflector system.

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Fig. 3 c. Obliquity $17^\circ.5$: setting for measurement of coma by adjustment to deflector system.

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Fig. 5. Obliquity $17^\circ.5$: setting for similar measurement of coma.

* cf. *loc. cit.* Figs. 1 and 6.

† In a recent criticism of the instrument as a means of test Mr Hasselkus has indicated the desirability of being able to compare the results of measurement with those of calculation.

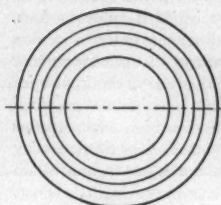


Fig. 1

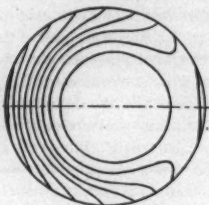


Fig. 2

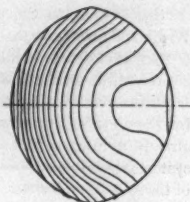


Fig. 3

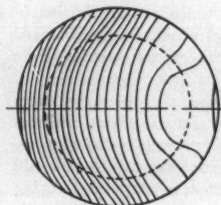


Fig. 1a

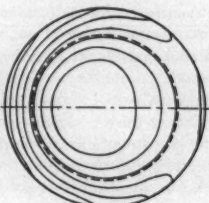


Fig. 2a

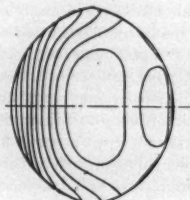


Fig. 3a

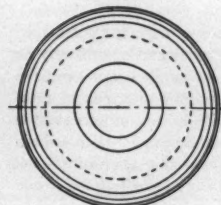


Fig. 4

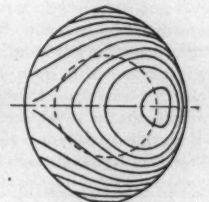


Fig. 3c

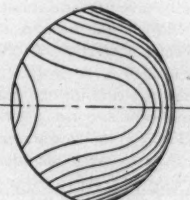


Fig. 3b

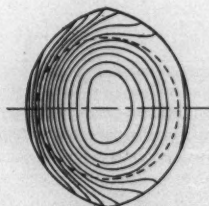


Fig. 5

a certain known deflection may be imparted, by means of a suitably arranged deflecting system, to the parallel beam of light originally incident upon the lens. Whilst the former method is preferable on account of the directness of the procedure of making a determination, the latter possesses the advantages that the adjustment is susceptible mechanically of more accurate control, and that the measuring device requisite for the purpose in view, viz. the deflector system*—originally provided for the annulment and simultaneous measurement of the distortion effect—forms already an integral part of the instrument. But as the extended use of the deflecting system is possible only when certain assumptions are made, relating to the perfection of the system under test, this method is to be regarded only as approximate, whereas the other method yields directly results free from direct error. In the following the mode of procedure to be followed in the quantitative determination of the aberrations will be found described and, to facilitate the direct application in practice, reference is made to contour figures (Figs. 1-5) which, being calculated for a particular instance of an $F/5$ anastigmat objective, represent the settings for a typical test.

Considering firstly the measurement of axial spherical aberration, we have accordingly to adjust the reference mirror longitudinally along the slide until the contour figure assumes for the zone in question (indicated in the figures by a broken line) the form shown in Fig. 4. Slight flexure of the instrument, by hand, will indicate to the observer the precise zonal adjustment effected, the path length being so varied as to cause a dark fringe to be evanescent at that certain zone. The longitudinal spherical aberration is then read from the micrometer adjustment to the reference mirror carriage.

Proceeding now to measure the oblique errors of the system, for extra-axial beams aberrations affecting the position of the image appear and must firstly be considered. A small deflection is imparted to the incident parallel beam and the carriage carrying the spherical reference mirror is adjusted from its normal automatically-adopted position longitudinally along the slide until on observation (Fig. 3 *a*) the lens is seen to be focussed axially in a secondary plane. Both adjustments are then recorded. We then derive the measure of this aberration—the distortion—regarded as an elongation $\delta y'$ to the radial image (o, y') at the image plane (x', y') as follows:

$$\frac{\delta y'}{y'} = \frac{2\delta\omega}{\epsilon} \sin 2\omega' \equiv \left\{ 200 \frac{\delta\omega}{\epsilon} \sin 2\omega' \right\} \text{ per cent.} \quad \dots\dots(1),$$

or alternatively:

$$\frac{\delta y'}{y'} = 100\delta\omega \left(\cot \omega + \frac{1}{\epsilon} \tan \omega \right) \text{ per cent.} \quad \dots\dots(1 a),$$

where ω is the obliquity of the incident beam to the optical axis of the system under test, $\delta\omega$ is the deviation caused by the deflector system, ϵ is the stop magnification (generally $\epsilon \neq 1$), and accented symbols refer to corresponding quantities in the image space of the system.

* F. Twyman, *Trans. Opt. Soc.* 22 (1920-21) 174.

The image curvature and astigmatism are next to be measured. This measurement is effected likewise by longitudinal adjustments to the spherical reference mirror. Such an adjustment having already been made for the secondary plane, and the corresponding longitudinal displacement recorded, a similar setting is made for the primary plane (Fig. 3 *b*). These measurements are made, of course, along the gaussian principal ray, considered here as identical with the chief ray of the convergent beam, and are in the form required, their projections upon the optical axis being readily determined.

To determine now what is in practice referred to generally as primary coma, settings (Fig. 5), similar to those made for the axial beam during the determination of spherical aberration, are to be repeated, and the longitudinal aberration along the principal ray will be obtained in precisely the same way.

This essentially completes the determination of the aberrations, considered as such, which are of predominant interest and importance to the computer and user of a photographic objective. Nevertheless it is considered desirable to include a test against fulfilment of the sine condition, non-compliance with which may be regarded, in the wider sense, as an aberration. In accordance with the usual practice when the beam incident upon the lens system is parallel, we have to determine the zonal variation of the focal length $f_\theta (= h/\sin \theta')$, h being the semi-diameter of the aperture of the incident parallel beam and θ' the inclination to the optical axis of rays emergent from that zone, for $\omega = 0$. In the derivation of this aberration, approximations are necessarily involved, as will be clear from the fact that a precise determination would require an exact knowledge of θ' . The estimation of the aberration in question proceeds by determining the comatic displacement $\delta y'$ at the image plane, for the sagittal section of the lens system. This may be measured by a transverse adjustment $\delta y'$ to the reference mirror, the optical system having been rotated from zero to a small known obliquity ($\omega = 0.04$ is generally a convenient obliquity to employ). The modified contour figure corresponding to this setting (Fig. 2 *a*) is to be such that the variation of the relative retardation Δ in the circular zone and at the point in question is zero (it is interesting to note that then, incidentally, variations in Δ dependent upon terms of second and higher orders in $\tan \omega$ being negligible, Δ is in general constant for that zone, as will be evident upon reference to the figure). Having thus determined $\delta y'$, and knowing y' in that ω is known, the zonal variation δf of the focal length f is estimated from the approximate relation

$$\frac{\delta f}{f} = \frac{\delta y'}{y'} - \frac{\delta s'}{ef} \left(1 - \frac{1}{8N^2} \right) \quad \dots\dots(2),$$

where the expression within the bracket is an approximation for $\cos \theta'$ in terms of the known aperture ratio N , and $\delta s'$ denotes the longitudinal spherical aberration for the particular zone, positive in the sense of under-correction.

If the deflector system be used for measuring the axial and oblique spherical aberration, the procedure is very similar to that described above. The determination is effected by imparting a certain small deviation $d\omega$ to the incident parallel beam, thereby exactly annulling the aberrational deviation for any particular restricted

region of the finally emergent beam. One has similarly in this case, as formerly remarked, to seek an adjustment as a result of which the relative retardation is caused to assume a stationary value at the point in question. However, before the corresponding effect upon the emergent convergent beam can be evaluated in terms of the required longitudinal aberration, the deflection $d\theta'$, correlate to $d\omega$ in the image space, must first be obtained. This cannot be done for any but small apertures without our introducing an arbitrary assumption concerning the image-forming properties of the lens system: the degree of approximation thus attained will be sufficient for most practically occurring cases, exception being made for very large aperture systems.

We assume now (1) that the optical system does not depart widely from fulfillment of the sine condition for any working obliquity, also (2) that the objective is working in air, and eventually (3) that the stop magnification $\epsilon \div 1$. Then as a first approximation it follows, in the former notation, that

$$\epsilon \frac{d\theta'}{d\omega} = \frac{\cos \theta'}{\cos \theta},$$

whence, for $\theta = 0$,

$$\delta\theta' = \frac{1}{\epsilon} \cos \theta' \cdot \delta\omega \quad \dots\dots(3),$$

the usual convention as to sign being adopted.

Such a small deviation $\delta\omega$ must be utilized to nullify the aberrational deviation $\delta\theta'$ of the emergent ray. This, to our approximation, is related to the longitudinal aberration δs through

$$\delta s' = \frac{\epsilon f}{\cos \omega} \cdot \frac{\delta\theta'}{\sin \theta'} = \frac{f}{\cos \omega} \cot \theta' \cdot \delta\omega \quad \dots\dots(4).$$

Thus the axial or oblique primary spherical aberration may be determined by finding, by means of the deflector system, the quantity $\delta\omega$ and substituting directly in equation (4). The quantity $\cot \theta'$, not being available directly, must as above be approximated to from N by virtue of assumption (1), which enables the expression of $\cot \theta'$ for the axial beam to be put in the form

$$\cot \theta' \div 2N \left(1 - \frac{1}{8N^2}\right).$$

For a finite ω the correction factor $\sec^2 \omega'$ must in addition be employed by reason of the reduction of the angular effective aperture consequent upon the polar recession of the gaussian image (to which the actual image is assumed to approximate) with the obliquity. Thus for measurement, by the deflector system, of axial spherical aberration (Fig. 1 a),

$$\frac{\delta s'}{f} = 2N \left(1 - \frac{1}{8N^2}\right) \cdot \delta\omega \quad \dots\dots(5)$$

and for measurement of primary oblique spherical aberration,

$$\frac{\delta s'}{f} = \frac{2N}{\cos^2 \omega} \left(1 - \frac{1}{8N^2}\right) \cdot \delta\omega \quad \dots\dots(6).$$

Likewise the deflector system may also be used in the estimation of the departure from fulfilment of the sine condition, the setting (Fig. 2 a) being made upon the same principle as before, the quantity $\delta y'$ in equation (2) being found from the determined deflection δy for the required zone and sagittal section of the lens by

$$\frac{\delta y'}{y'} = \frac{\epsilon \delta z}{\tan \omega} = \delta \omega \cot \omega.$$

Such determinations being now made for certain chosen obliquities and for radiations of certain chosen wave-lengths, sufficient data will be accumulated to allow of the graphical representation of the aberrations of the tested lens system in the usual way, viz., by plotting the aberrations with aperture and obliquity against these quantities respectively. The results of an actual test*, together with the results

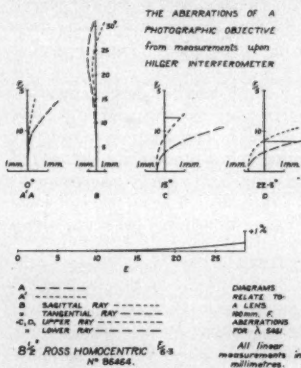


Fig. 6.

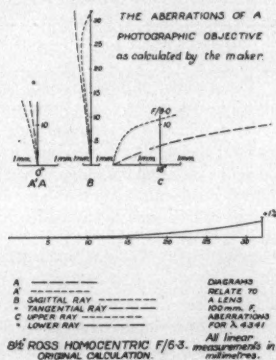


Fig. 7.

obtained by calculation for the system in question, are reproduced in Figs. 6 and 7, where

- curve A represents the spherical aberration,
- " A' " " zonal variation in focal length,
- " B " " image curvature and astigmatism,
- " C, D " " primary coma at stated obliquities,
- " E " " distortion,

all linear dimensions being in millimetres.

The light sources available for such tests are (1) the mercury vapour lamp, (2) the hydrogen vacuum tube, and (3) the recently introduced neon discharge lamp.

* Grateful acknowledgement is due to Messrs Ross, Ltd., who, through the intermedium of the British Scientific Instrument Research Association, have been kind enough to place at the disposal of Messrs Adam Hilger, Ltd., for this purpose the Ross "Homocentric" lens detailed in the figures.

Using such sources of illumination, measurements may be made for the following wave-lengths:

- (1) (5791, 5770), 5461, 4359 A.U.
- (2) 6563, 4861, 4341 A.U.
- (3) 5853 A.U.

Photographic objectives will generally be required to be tested at least for 4341 and 5893 A.U., these wave-lengths generally sufficing also for the determination of the chromatic aberration. The former case is readily suited and presents no difficulty: in the majority of such instances it will be sufficient to carry out the tests using the blue Hg radiation 4359 A.U. The latter case is met practically by the use of the neon lamp. Otherwise, considering the facility and convenience which would be gained thereby, the blue and yellow Hg radiations might well be allowed general preference to those of hydrogen of neighbouring wave-lengths, except under those circumstances where the heterogeneity of the former radiations is not of negligible effect.

The accuracy of measurements conducted as above described depends upon the precision with which single settings can be effected and recorded, and, in the case of the deflection measurements, the extent to which the assumptions made are justified: the slight errors arising from assuming unit stop magnification may, of course, be obviated if ϵ , differing slightly from unity, be known, by taking account of it when substituting in the above formulae. Practically it is found that individual settings can be repeated consistently in most cases to within 3 per cent., with a lower limit of $\delta(\delta\epsilon') = \cdot 025$ mm.

When testing telephoto objectives it will be necessary to observe certain modifications in procedure. Firstly, definite account of ϵ must obviously be taken: failing an exact knowledge or a precise determination of this particular, one may use as an approximate value for $1/\epsilon$ the generally-known telephoto magnification μ , which value should be used when evaluating where necessary. Furthermore, since the first adjustment is no longer made along the chief ray of the emergent convergent beam, longitudinal adjustments of the spherical reference mirror from the normal position must in general be accompanied by small transverse displacements. The proper direction and approximate amount of such small displacements will be rendered evident by mental comparison of the observed contour image with the appearance sought.

In conclusion I would gladly record my indebtedness to Mr F. Twyman, the director of the above work, for useful suggestions and pecuniary aid in connection with the above and allied problems.

Note added later. Attention must be called to a point of difference between the results of measurement and of calculation as recorded in Figs. 6 and 7, the latter of which were not available to the writer until shortly before the meeting. The calculations were made, according to general computing practice, for the hydrogen line 4341 A.U., whereas the measurements were made for the mercury line 5461 A.U.

DISCUSSION

Mr J. W. Hasselkus: I have not much to add to Mr Perry's excellent paper, except perhaps to draw attention to the very close resemblance between the results shown by the measurements obtained on the interferometer and those of calculation. The great simplicity is particularly interesting, having regard to the fact that the type of lens chosen for the experiments is one which was introduced twenty-four years ago, having been patented in 1900. The calculated curves* are those of the original design, while the observed curves are for a lens made to the same design as late as 1918. These curves naturally differ considerably from those which have been obtained with more modern types of lenses.

With regard to the interpretation of the various curves shown, I need only say that they are based on the same scale and method as were originally suggested by Dr P. Rudolph and employed in the *History of the Photographic Objective* by M. von Rohr. The only addition to the curves shown in this book are those referring to the coma, or spherical aberration of oblique rays. These are drawn up in the same manner as those for the central spherical aberration, but naturally develop into two curves, one for the half of the aperture above the principal ray and the other for the half of the aperture below the principal ray.

Mr T. Smith: The author is to be congratulated on solving a problem likely to be of distinct benefit to the optical industry of this country. It is of interest to observe how the standard of performance required of anastigmat lenses has straitened since the time when the lens was designed of which the test results have been exhibited*.

Dr L. C. Martin: I should like to thank Mr Perry for his paper and to express the hope that he will amplify, to some extent, his description of experimental details. While it is doubtless comparatively simple to obtain a measurement of the aberrations in the case of such systems as a photographic lens, where the errors of the wave-surface are comparatively large, I have found it by no means easy to obtain adequate numerical measurements for spherical aberration on the simple lens-testing interferometer when working with telescope objectives. I have, however, with the help of some of my students, been using the interferometer for measurement of chromatic aberration and on the whole I think this test is one of the most reliable, for that purpose, of which I know. The test for spherical aberration is quite another matter, as the whole absolute accuracy depends on the perfection of figure of the convex reflectors supplied by the makers, and although the excellence of Messrs Hilger's work is beyond question, it is well known that convex surfaces of perfect figure are by no means easy to obtain. Of course the majority of telescope objectives do in practice exhibit irregularities of figure which are at least as serious as the true residual aberrations, and for the detection of these the interferometer is invaluable, since it yields results not readily obtained by any other method. I should

* The curves referred to were shown at the meeting, but are not given in this paper; Fig. 7 illustrates the results of calculation for a more modern type of lens.

be very interested to know what is the least amount of spherical aberration expressed in terms of departure of a wave-surface from true sphericity, which Mr Perry considers to be measurable by the interferometer.

Mr Perry: I have to express my thanks for the appreciative remarks in regard to the work and results recorded.

In reply to Dr Martin, it must be admitted that convex mirrors of perfect figure are not easy to obtain. The instrument in question affords in itself a means whereby the approach to perfection may be estimated and thus be carried to the necessary limits. In regard to the precision attainable in measuring for any point the departure from true sphericity of wave-surface, which may be measured in terms of the linear deformation along the wave-normal at that point, the means employed allows of observing readily a deformation amounting to $\frac{1}{4}\lambda$; the significance of such a measurement at the limit of precision is measured by the degree to which absolute perfection exists in the interferometer system. This is governed by many factors, some of which, e.g. the incidental heterogeneous effects of temperature variation and mechanical strain, are under the control of the observer, and will here be ignored. Since the shortcomings inherent in any system may be demonstrated and eliminated to the limit above mentioned, such small departures from sphericity may be stated accurately to $\pm \frac{1}{4}\lambda$.

THE RELATION BETWEEN APERTURE, AXIAL THICKNESS, AND FORM FOR A SINGLE LENS

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MS. received, 13th December, 1923. Read and discussed, 10th January, 1924.

ABSTRACT. A comparison is made between the exact and various approximate expressions connecting the attainable aperture and the axial thickness for simple lenses of various shapes.

Introductory

THE determination of the thicknesses to be given to lenses of known shapes to enable a given aperture to be attained is a problem which occurs frequently in computing lens systems. When ray tracing is the method of design employed any convenient solution may be used—for example, the direct graphical method of drawing a section of the lens on a convenient scale with the required curvatures, the centre of the second surface being found by striking intersecting arcs of the given radius from points on the first circle whose distance apart is equal to the assigned aperture, and measuring the axial thickness on this diagram. If algebraic methods of calculation are used, an algebraic solution of the problem is necessary. Such solutions assume the simplest form in connection with the converse problem of finding the maximum aperture which is attainable with a given thickness. From the point of view of the development of algebraic methods it happens that this simpler converse problem is unimportant, but solutions, both exact and approximate, may be used as a means of constructing solutions to the real problem. In all suitable expressions the square of the aperture, not its first power, is involved; it is convenient to consider the semi-aperture rather than the whole diameter, and to represent its square by a simple symbol, say a . Thus πa represents the area of the aperture opening, the quantity of photometric significance.

The exact formula

Since two spheres meet only in a single real circle, which divides each sphere into two parts, the exact formula for the aperture is the solution for the four separate lenses obtained by combining the two alternative portions of each sphere. Thus if the first surface with its centre at C_1 meets the axis at A_1 and B_1 , and the second surface with its centre at C_2 meets it at A_2 and B_2 , the aperture is the same whether the vertices of the refracting segments are A_1 and A_2 , A_1 and B_2 , B_1 and A_2 , or

B_1 and B_2 . This in itself indicates that the exact formula will be inconveniently complex in form, and that, though a linear equation is obtained for a when the thickness is known, the equation is a biquadratic when the thickness is the unknown variable. It is not difficult to prove that the aperture is in fact given by

$$4a(C_1C_2)^2 = A_1A_2 \cdot A_1B_2 \cdot A_2B_1 \cdot B_1B_2.$$

In terms of a particular thickness, say A_1A_2 , which may be denoted by t , the formula becomes

$$4a(R_1 - R_2 + tR_1R_2)^2 = t(2 - tR_1)(2 + tR_2)(2R_1 - 2R_2 + tR_1R_2) \dots (1),$$

where R_1 and R_2 are the curvatures of the two surfaces*. This relation may be expressed in a non-dimensional form by taking new variables A , T , and X , ratios depending respectively on the aperture, the thickness, and the lens shape in relation to the total lens curvature, the definitions being

$$\left. \begin{aligned} A &= \frac{1}{2}a(R_1 - R_2)^2, & T &= \frac{1}{2}t(R_1 - R_2) \\ X &= \frac{4R_1R_2}{(R_1 - R_2)^2} = \left(\frac{R_1 + R_2}{R_1 - R_2}\right)^2 - 1 \end{aligned} \right\} \dots (2).$$

In terms of these quantities (1) becomes

$$A(1 + bX) = b - b^2 \dots (3),$$

where

$$b = T(1 + \frac{1}{4}TX) \dots (4).$$

Equations (3) and (4) show that the biquadratic (1) can be solved in a form suitable for numerical computation, the solution being

$$T = \frac{2}{X} \{ (1 + bX)^{\frac{1}{2}} - 1 \} = \frac{2b}{(1 + bX)^{\frac{1}{2}} + 1} \dots (5),$$

where b must now be obtained from

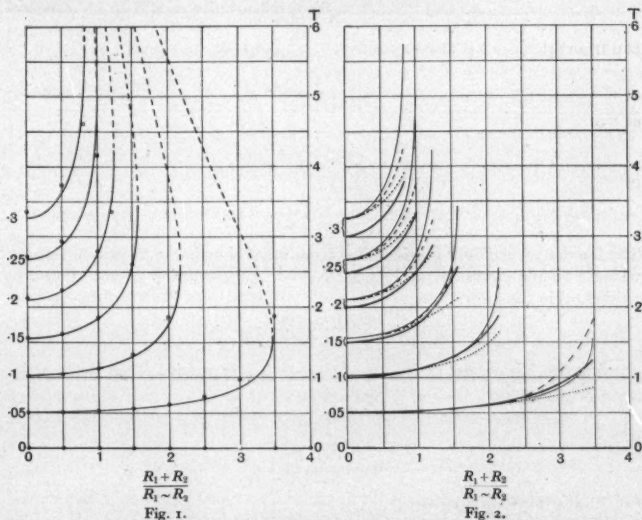
$$\left. \begin{aligned} 2b &= 1 - AX - \{(1 - AX)^2 - 4A\}^{\frac{1}{2}} \\ \text{or} \quad b &= \frac{2A}{1 - AX + \{(1 - AX)^2 - 4A\}^{\frac{1}{2}}} \end{aligned} \right\} \dots (6).$$

For numerical computation the second of these alternative forms is preferable in each case. The square roots occurring in these equations may of course carry either sign. The expressions as written above correspond to the case most frequently encountered, in which neither surface exceeds half the complete sphere, if each square root is assigned a positive value. This solution, for which the aperture and the thickness vanish together, applies to apertures corresponding to all values of A not exceeding the smaller root of the equation

$$(1 - AX)^2 = 4A \dots (7).$$

* The usual convention, that surfaces convex to the incident light have positive curvature, is followed. This must be applied to the particular case in which the separation of the vertices of the surfaces is t . The curvature signs for any of the other three cases are of course different.

Fig. 1 shows the variation of T for values of $\frac{R_1 + R_2}{R_1 - R_2}$ from 0 to ± 4 when A varies from 0 to $\cdot 3$ in steps of $\cdot 05^*$. The portions of the curves which correspond to the usual case are shown by continuous lines; beyond the limit given by (7) broken lines are used. As an indication of the scale it may be noted that for the equiconvex form the aperture and the focal length are approximately equal in the neighbourhood of $A = \cdot 25$, so that the curves shown suffice to provide an adequate test of any simplified formulae by direct comparison with their graphs.



NOTE:—The figures to the left of each diagram denote the values of A for the adjacent curves.

Approximate formulae

The only well-known formula connecting the aperture and thickness states that “the semi-aperture is a mean proportional between the axial thickness and the focal length.” This is based on the assumption that the refractive index of a convex lens is approximately 1.5, and in the notation of the previous section takes the form

$$T \simeq A \quad \dots(8).$$

That such a formula is entirely inadequate, and for meniscus lenses very misleading, is evident by comparing the straight lines $T = \text{constant}$ marked on Fig. 1

* These correspond roughly to relative apertures having the following f /numbers: $A = 0, f/\infty$; $A = 0.05, f/20$; $A = 0.1, f/16$; $A = 0.15, f/13$; $A = 0.2, f/11$; $A = 0.25, f/10$; $A = 0.3, f/9$.

with the curves derived from the exact formula. This approximation errs on the undesirable side in that it invariably yields an underestimate for the axial thickness.

From the exact expressions which have been given, inequalities may be constructed which lead to other approximate expressions of interest. These, to be of practical importance, must give values of T increasing with X . Examples of the type which give a one to one correspondence between A and T may be considered. First taking a case in which an inferior limit to T is obtained, equation (3) gives

$$b > \frac{A}{1 - AX}$$

and from (4), since b is always positive,

$$\frac{T}{1 - \frac{1}{4}TX} > b \quad \dots(9),$$

so that

$$\frac{T}{1 - \frac{1}{4}TX} > \frac{A}{1 - AX}$$

or

$$T \triangleq \frac{A}{1 - \frac{3}{4}AX} \quad \dots(10).$$

The fraction on the right is evidently a crude approximation to T , as is obvious by taking a simple case such as $X = 0$. A closer approximation may be found by writing (3) in the form

$$\frac{A}{b} = \frac{1 - b}{1 + bX}$$

showing that b is at least as great as A since $X + 1$ is positive. Thus the equation gives the inequality

$$A(1 + bX) < b(1 - A)$$

or

$$b > \frac{A}{1 - A(X + 1)} \quad \dots(11).$$

Combining this with (9) gives

$$T \triangleq \frac{A}{1 - A(\frac{1}{4}X + 1)} \quad \dots(12).$$

The introduction of the additional term compared with (10) has improved the approximation greatly, but the variation with X remains insufficiently rapid. Fig. 2 (dotted lines) shows this formula plotted for the same values of A as are used in Fig. 1.

The expression (12) is the simplest example of its type which can be derived by the direct use of inequalities. A closer fit can be obtained by empirical modifications, as, for example, by neglecting the difference between b and T , so that the expression on the right of (11) becomes an approximation value for T . As would be expected, this sometimes gives too high and sometimes too low a value of T . For a considerable region of much practical importance this formula is sufficiently exact.

Fig. 2 shows the comparison between these approximate values (thick lines) and the exact values (thin lines).

It is desirable, however, to construct other expressions which will err in the other direction. In these the formula will either involve the shape of the lens in the numerator of the approximation as well as the denominator, or alternatively the denominator will no longer be linear. The simplest method of constructing such expressions is to expand the square roots in the exact formula by the method of continued fractions and thence derive the successive convergents. It will only be necessary to go one step beyond that taken in the previous example to obtain a suitable solution.

From (5), by the method indicated,

$$T \triangleq \frac{b(1 + \frac{1}{2}bX)}{1 + \frac{1}{2}bX}.$$

In the factors involving X the value of b given by (11) may be substituted, leading to

$$T \triangleq \frac{b\{1 - A(\frac{3}{2}X + 1)\}}{1 - A(\frac{1}{2}X + 1)} \quad \dots(13).$$

If the same value is substituted for b in the remaining factor, the expression becomes

$$T \triangleq \frac{A\{1 - A(\frac{3}{2}X + 1)\}}{(1 - A(X + 1))\{1 - A(\frac{1}{2}X + 1)\}}.$$

This is approximately equivalent to

$$T \triangleq \frac{A\{1 - A(\frac{3}{2}X + 1)\}}{1 - 2A(\frac{3}{2}X + 1)} \quad \dots(14).$$

The curves (broken lines) derived from this expression are given in Fig. 2. The approximation can be carried to a higher stage, if desired, by obtaining a closer relation between b and A . From equation (6) the next approximation is

$$b \triangleq \frac{A\{1 - A(X + 2)\}}{1 - A(2X + 3) + A^2(X + 1)^2}.$$

The substitution of this value for b in any of the foregoing expressions would lead to formulae too complex to be of much value. It may be simplified by the omission of the last term in the denominator, and then leads by (13) to

$$T \triangleq \frac{A\{1 - A(X + 2)\}\{1 - A(\frac{3}{2}X + 1)\}}{(1 - A(2X + 3))\{1 - A(\frac{1}{2}X + 1)\}}$$

or approximately

$$T \triangleq \frac{A\{1 - A(\frac{7}{2}X + 3)\}}{1 - A(\frac{5}{2}X + 4)}.$$

This expression is less convenient than (14), and owing to the omission of denominator terms it gives decidedly exaggerated values for T . A comparison of the two suggests a form more accurate than either, viz.

$$T \triangleq \frac{A(4 - 2A(X + 3))}{4 - 5A(X + 2)} \quad \dots(15).$$

The points marked on Fig. 1 by circles have been calculated from this expression using the same values of A as for the curves. It will be seen that the formula errs on the side of giving large values of T , that is in the desirable direction. It is only for large values of A , with one of the surfaces almost a hemisphere, that T is underestimated, and these cases are of extreme rarity in optical instruments. This formula should be found suitable for all ordinary purposes. Whenever the marginal thickness is to be accurately zero, it is of course necessary to use the exact formula (5) and (6).

DISCUSSION

Mr J. W. Perry: Computers will be glad to have available the expression for the axial thickness in the convenient and accurate form given by (15). For the majority of present day practical purposes it is sufficiently approximate, and much more convenient, simply to use in conjunction with the first approximation term an additive correcting term in the numerator, thus (using Mr Smith's notation),

$$T \triangleq A + \frac{A^2}{3}(3x^2 + 1) = A \left(1 + \frac{A}{3} \right) + A^2 x^2,$$

where $x = \frac{R_1 + R_2}{R_1 - R_2}$. T as so calculated is slightly in excess for normal lens forms, correct for a certain x , and in defect for more extreme forms. The slight errors arising from the use of this expression may then, if not quite negligible, easily be corrected upon the completion of the analytical work, with the net result of a saving of time on the whole computation.

Mr Smith: It is interesting to hear that Mr Perry has found a two-term formula useful for computing thicknesses. His formula seems well adapted for moderate apertures, and agrees with (15) when $5A(x^2 + 1) = 1$. Possibly a relation of this kind might be taken as a limiting definition of what may be regarded as moderate apertures. For greater apertures than this the estimate it gives rapidly becomes distinctly small, as indeed is the case with any simple formula which has not a decreasing denominator. The use of a correcting term to an expression is more common than that of a correcting factor, but in principle the one does not seem to be either more or less desirable than the other.

A NEW FORM OF CORNEAL MICROSCOPE WITH COMBINED SLIT LAMP ILLUMINATING DEVICE

By E. F. FINCHAM

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MS. received, 1st February, 1924. Read and discussed, 14th February, 1924.

ABSTRACT. For the clinical examination of the cornea and anterior portions of the interior of the living eye, a procedure often necessary, a specially mounted microscope, monocular or binocular, is used in conjunction with a so-called Slit Lamp. In the apparatus as used at present two instruments are separately mounted and must be adjusted independently. This paper, which is the outcome of some experiments made in the Department of Applied Optics of the Northampton Polytechnic Institute, London, upon the microscopic examination of the living eye, describes a combined instrument in which the adjustment is common to both the microscope and the lamp, thus greatly facilitating the observation.

Introductory

In the routine examination of the eye it is necessary to study, not only the condition of the fundus by means of the ophthalmoscope, but also the more anterior portions such as the cornea, iris, and crystalline lens. Certain pathological conditions produce changes therein and a critical examination materially assists the diagnosis. In order to carry out this examination, it is necessary firstly to illuminate the part and secondly to observe it under magnification.

In the past the illumination was obtained by focussing on to the part to be examined the light from a convenient source, by means of a condensing lens held in the hand, while the observation was made with a hand magnifier called a corneal loupe. A binocular loupe, worn in the form of spectacles by the person making the examination, is sometimes used.

For some years now the corneal microscope, consisting of a monocular or binocular instrument having a fairly long working distance and a convenient mounting, has superseded the corneal loupe for the purpose of this examination.

The introduction of the slit lamp, devised by Gullstrand and made by Messrs Zeiss, as a means of illuminating the eye, has made possible the microscopic examination of the transparent ocular media in section. So important has this method of clinical examination become in demonstrating the earliest stages of many ocular diseases that complete atlases of the conditions observed, and descriptive works on the technique of the microscopic examination of the living eye, have been published by Vogt, Koeppe, and others.

Principle of the slit lamp

The cornea, crystalline lens, and vitreous humour, although transparent, produce by reason of their structure a certain amount of scattering of the light falling upon them. This scattering is sufficient, if the illumination be intense and critical, to render visible the structure of these apparently transparent tissues.

The function of the slit lamp illuminator is to produce upon the part under observation an intensely bright and well-defined image of an illuminated slit, thus illuminating a section of the tissues which can readily be examined through the surrounding transparent structures; these latter, not being illuminated, will in general be invisible. The illuminated section will then be seen standing out brightly against a perfectly black background—its thickness being determined by the width of the slit image.

Fig. 1 represents the anterior portion of the eye and shows the image of the slit illuminating a section of the cornea. The side of the eye has been removed to show a cut section of the cornea for comparison.

Just as it is important that a section produced by a microtome should be thin and that the knife with which it is cut should be sharp, so to ensure good resolution of detail in the section produced by this optical means, the image of the slit must be narrow and its definition as good as possible. In order to secure this condition, it will be seen that it becomes necessary to keep the observing microscope directed towards the exact focus of the illuminating beam, as only at this point will the section be thin and clean cut.

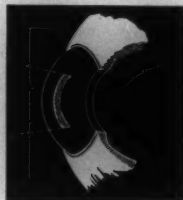


Fig. 1

Fixed central mounting in new instrument

Hitherto, the illuminating system and the observing microscope have been free to move independently; this necessitated the readjustment of both direction and focus when the inspection was moved from one part of the eye to another. In the present instrument, both the illuminating system and the microscope are mounted upon an arc which has its centre at the focus of the illuminating beam (Fig. 2). Thus the only adjustment that is required with this instrument is the focussing of the projected image upon the part of the eye to be examined, when the image of the latter will automatically be brought into the centre of the field of the microscope. This adjustment is made by racking forward the arc carrying the two components of the instrument—exact focus being secured when the image appears centrally in the microscope field. The decentering effect produced on the microscope image when the instrument is out of focus is shown in Fig. 2. The cornea of the eye, as shown with a broken line, is placed beyond the focus of the illuminating beam and is consequently out of centre with the microscope. In order to examine the section under various angles, it is only necessary to swing the microscope along the arc, the centering and focussing remaining unchanged. This movement is necessary as,

although where possible it is often desirable to examine the section perpendicularly to the illuminating beam, a smaller angle of observation must be employed when observing the deeper structures, owing to the oblique view being cut off by the iris.

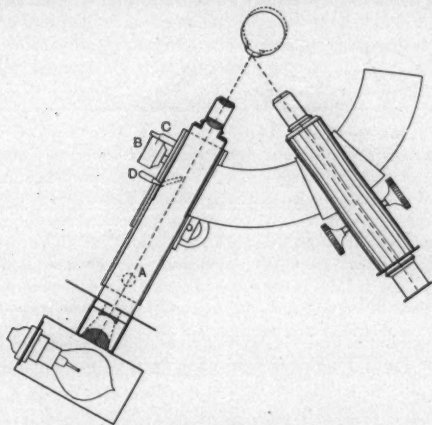


Fig. 2

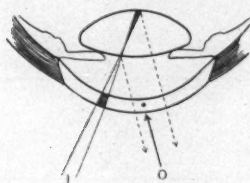
Independent movement of illuminating system

Although, for the purpose of studying the sections of the optical media, the combined centering of microscope and illuminating system is the ideal arrangement, it is found in practice that many pathological states of the eye are seen to greater advantage by indirect methods of illumination which could not be obtained under these conditions.

Fig. 3 represents the anterior portion of an eye having a small and faint opacity situated in the cornea. If the illuminating beam be directed through the cornea to one side of the disturbance so that the crystalline lens be illuminated, then by adjusting the direction of observation correctly the opacity will be seen more or less dark against the bright background of the lens. Under these conditions the density of the opacity can be estimated by comparing its brightness with that of the background.

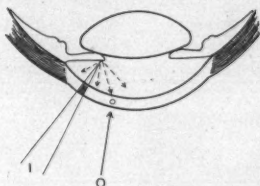
It is sometimes possible to illuminate such an opacity and observe it against a dark background, by arranging the directions of observation and illumination as in Fig. 4. In this case the beam is concentrated upon the iris from which, if it be pale in colour, sufficient light is scattered to illuminate the small white or translucent body in the cornea. The microscope is then directed so that the opacity illuminated in this manner is seen against the dark background of the pupil.

The many possible methods of making a microscopical examination of the eye under different conditions of illumination have been exhaustively dealt with by Graves in a recent paper published by the Ophthalmological Society, to which I would refer all who are interested in ophthalmic apparatus of this nature*.



1 Illuminating beam

Fig. 3



O Direction of observation

Fig. 4

Optical construction of slit lamp

The illuminant in this model is a 12-volt, 24-watt, projection-type lamp, having a small straight filament which is particularly suitable for illuminating the slit aperture used. Between this and the slit a condensing system of short focal length, consisting of a steep hemispherical positive followed by a strong negative lens, is placed so as to produce a practically parallel beam just large enough to cover the length of the slit. This device was introduced in order that a maximum amount of the light passing the slit should enter the projection lens, which is of comparatively small aperture.

The projected image is produced by a 2-inch microscope objective placed at a distance of 160 mm. from the slit. This is found to give a well-defined image sufficiently free from aberration effects. In the plate carrying the slit is also a small circular aperture which may be substituted for the slit when it is desired to examine surfaces, either of the optical media or of the iris, conjunctiva, etc. By suitable adjustment of the directions of illumination and observation when using this aperture, the tessellated appearance of the corneal endothelium and in some cases the stellate arrangement of the lens fibres are made visible. The microscope for ordinary use gives a magnification of about 20 diameters and although this of course may be increased, it is not generally useful to employ more than double this magnification.

To allow of the adjustments for indirect illumination which have been described, the illuminating system has been mounted upon a plate which is capable of rotation in a horizontal direction about a bearing *A* (Fig. 2) by means of the screw *B*. In order that the original position of correct centering relative to the microscope may be returned to without difficulty, the head of the driving screw is marked with an index line to read against a zero mark on the pointer *C*.

* Basil Graves, "Microscopy of the Living Eye." *Trans. Ophth. Soc.* 43 (1923) 387.

In addition to this, the projection lens has been mounted in a sliding tube operated by the handle *D*, to enable the focussed image of the slit to be projected, to a point behind that under observation in the microscope, if desired.

In this way the possibility of indirect illumination has been gained without sacrificing the principle of the combined adjustment of illuminating system and microscope, which for many purposes, and especially that of making measurements of intra-ocular distances, is a great advantage.

Measurement of intra-ocular distances

An important use of the corneal microscope is the determination of the apparent depth of the anterior chamber of the eye, a measurement of considerable value, not only in experimental work, but also to the ophthalmic surgeon in the diagnosis of glaucoma. This is made with the existing forms of corneal microscopes by determining the adjustment necessary in order to focus successively the cornea and the anterior surface of the crystalline lens. This method is, however, open to error as it depends upon the observer's estimation of exact focus and also upon the control of his own accommodation. These difficulties are avoided in making the measurement with the new instrument. Owing to the fact that the microscope is directed constantly to the focus of the illuminating beam, the observed surface will appear central in the microscope field only when the light is accurately focussed upon it. It is therefore only necessary to measure the amount of movement required to bring successively the posterior surface of the cornea and the anterior surface of the lens into coincidence with a central cross-wire in the field of the microscope, to obtain the apparent distance between these two surfaces. To enable these readings to be taken, a small scale and vernier have been fitted to the focussing slide of the instrument.

Photography of the interior of the living eye

It was thought that a means of photographing the anterior portions of the eye under the illumination of the slit lamp would be of value in recording the state of pathological conditions, such as corneal ulcers and opacities in the ocular media.

The visual instrument was found to be unsuitable for this purpose as the illumination is not sufficiently intense to allow of instantaneous exposures being made. Some exposures were made, however, with a temporary apparatus using an arc lamp as the illuminant and a large slit, the image of which was very much reduced so that the maximum illumination was obtained. The photographs secured were not satisfactory, owing chiefly to the difficulty of maintaining perfect focus of camera and projection lens, good illumination, and steadiness of the patient, all at the moment of making the exposure.

To obviate these difficulties, a form of reflex camera was constructed so that the image could be watched and the focus and illumination adjusted up to the moment of exposure. Attention was then directed to the crystalline lens with the view to studying by this means that still unsolved problem, the mechanism of the accommodation. Some of the results obtained show a great deal of the structure

of the lens of the living eye (see Fig. 5); the relative curves of the cortex and the nucleus are plainly seen.

Unfortunately further work in this direction has been delayed, partly owing to the fact that in order to observe sufficient of the lens the pupil must be artificially dilated, while at the same time an active accommodation is required. If these conditions can be obtained or a young subject having a fixed large pupil can be found, the work will be continued in the hope of obtaining some interesting information on the changes which take place in the crystalline lens during the action of accommodation. In the meantime, other methods of attacking this problem are being pursued.

In conclusion I wish to express my indebtedness to my colleague Mr T. M. Rose for many helpful suggestions regarding the mechanical design of the corneal microscope and also for constructing the experimental model exhibited, and to Mr Basil Graves for much helpful criticism.

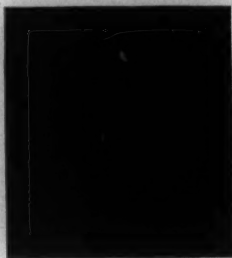


Fig. 5. Photograph of the crystalline lens in the living eye. $\times 4$.

DISCUSSION

Mr H. H. Emsley: Having had very little to do with the production of this instrument, I can say with some freedom that I think both Mr Fincham and Mr Rose should be congratulated on having produced the instrument at all in these days of vanishing equipment grants.

The subject of inspecting the eye, especially in a quantitative way, is not easy. If it be required to make measurements on the intra-ocular dimensions, either for the purpose of checking the previous measurements of Helmholtz, Listing, Gullstrand, etc. with modified apparatus, or as a necessary step towards further experimental work, one might use dead eyes if they can be obtained. When the difficulties of mounting and treating the dead parts are overcome, the measurements obtained are, after all, open to the objection that they may be different in the living eye.

Work on the living eye is more satisfactory, but here again there are difficulties. Firstly, the patient is inconvenienced by the brightness of the beam it is necessary to project into his eye, and secondly, in making measurements on such dimensions as the depth of the aqueous humour or the radii of the crystalline lens, we are observing through a system—the cornea—whose power we do not know for the particular eye under observation; the image space containing the quantity to be measured has a refractive index we do not know. If these unknown quantities can be measured, the process of calculation is lengthy.

Even if substances be injected to paralyse certain muscles and so render a part stationary, we are not observing the eye in its natural, free, elastic state.

Measurements are at present being made on young eyes with this instrument in an attempt to gather information that may help to a better understanding of the process of accommodation for near objects; but, because of such difficulties as those mentioned and others, the process is slow.

Lt.-Col. J. W. Gifford: Mr Fincham has been speaking of the difficulties of fixing the various constituents of the human eye before post-mortem changes set in. I have been wondering if it would be possible to inject the capillaries of the eye from the principal channel of blood supply, with formaline or some other suitable fixative, immediately after the eye has been removed.

Some five-and-twenty years ago I was successful in applying similar treatment to frogs and kittens. The method was to kill the animal by shock, then open up the cavity of the chest, sever the aorta, tie in a canula and connect it with a Wolff's bottle containing the fixative. A convenient series of shelves enabled me to regulate the pressure from one bottle. Formaline was then unknown, and the fixative used was a half per cent. solution of chromic acid. After the fourth kitten had been so treated unsuccessfully, the tongue of the fifth turned yellow and showed me that the fixative had got through. The animal was then dismembered, treated with an ascending series of alcohols, then with chloroform, and embedded in solid paraffin. Sections were then cut, stained, and mounted as usual. They are still in my possession, and are the best I ever made. The premature "rigor" was the difficulty, but did not occur in the case of the frog. I think that formaline, if injected warm, would not cause this at so early a stage, but I am not perhaps competent to judge.

Prof. Cheshire: Mr Fincham is to be complimented upon the simple way in which he has succeeded in linking together the illuminating apparatus and the observing microscope. Simple and effective as it is, I am not surprised to hear that it is entirely novel. I would suggest that the arrangement would lend itself admirably to the use of a binocular observing instrument, such as the Greenough microscope.

Comdr. N. Langlands: Is it not important to have the slit image free from aberration? In the Zeiss form of Gullstrand slit lamp this was achieved by employing figured surfaces.

In viewing an object by indirect illumination an iris diaphragm inside the viewing microscope might prove useful. The image of this stop would lie clear of the slit image in the eye and the direct illumination would be cut off.

When the angle between the microscope and the entering beam is nearly a right angle, is the light scattered from the tissues polarized? If that were the case, the use of a Nicol analyser might assist in defining the structure.

I have found that since spare parts for Nernst lamps became unobtainable these lamps could be run very successfully without either balances or heaters if a variable resistance were used in series with the filament.

Mr T. Smith: It should be unnecessary to pay great attention to the form of the condenser.

The discomfort of a bright light to the eye could be combined with better conditions for photography by using a bright source emitting radiation in the extreme

visible violet or slightly into the ultra-violet. Since the maximum sensitiveness of ordinary photographic plates is in the neighbourhood of the *G'* line this should be possible. Slight fluorescence of the crystalline lens and cornea, if present, should be advantageous rather than the reverse.

Mr G. W. Gosling: With reference to Mr Fincham's remarks on the question of measuring the depth of the anterior chamber, I should like to mention that the present method is to use an Ulbrich drum. This drum is graduated in millimetres and is fitted to one side of the rackwork, on the body of the binocular corneal microscope. Dr H. Hartinger* published a paper on the method, and the actual depths are computed from the apparent depths as given by the drum. The author took Gullstrand's schematic eye as his standard, and used the values for thicknesses and refractive indices as given by Gullstrand. I shall be pleased to send any reprints of this interesting paper to members who are interested.

One or two points occur to me to be worthy of discussion, in respect of the design of the instrument exhibited.

Why has the monocular observing microscope been fitted? The binocular form has distinct advantages, as the various tissues of the eye are seen in relief; this stereoscopic effect is particularly useful in helping the diagnosis of certain pathological changes and conditions.

When examining the deeper parts of the vitreous humour, it is essential that the beam of light from the slit lamp should be as nearly as possible on the axis of the observation objective. It seems to me that in Mr Fincham's instrument the design does not allow of very acute angles between the illuminating and observing systems.

Again, the position of the microscope cannot be altered relatively to the illuminating tube; thus one is confined to the temporal side of the left eye and the nasal side of the right eye.

With reference to Mr Fincham's remark on his trying a reflex type of camera for photography, I would like to point out that a stereoscopic reflex camera for taking photographs of the bulbous was shown at the Jena Ophthalmological Congress of 1922. The instrument was made by Messrs Zeiss for Lenz of Breslau. For all ordinary stereoscopic work the Drüner stereoscopic camera can be used; this allows all parts of the cornea, iris, and lens to be photographed, using the paired objectives of the binocular corneal microscope. Naturally an arc lamp is necessary as a source of illumination.

Comdr. Langlands has raised the point of trouble being experienced from polarizing effects. This opens up a very interesting question for research. Dr Koeppe† of Halle designed a special polarizing microscope for this work with the slit lamp, and he claims that certain histological and pathological conditions are shown by this means that cannot be seen by any other method.

* H. Hartinger, "Zur Messung der Kammertiefe und des Irisdurchmessers." *Z. f. Ophth. Opt.* 9 (1921) 135.

† L. Koeppe, *Die Ultra- und Polarisations Mikroskopische Erforschung des lebenden Auges und ihre Ergebnisse* (Leipzig: Ernst Bircher, 1921).

Mr Fincham: I should like to thank Col. Gifford for his remarks regarding the fixation of the eye and other tissues by the intra-arterial application of the fixing solution. This should prove a very valuable method of fixing when it is required to cut microscopic sections of the eye. Unfortunately I have not been able to procure human eyes sufficiently soon after removal to apply this method. Even if this procedure were employed and great care were taken to ensure the fixative being isotonic, it is doubtful whether one could guarantee the absence of artifacts in the nature of shrinkage or distortion of the optical surfaces. Also the action of a fixing solution invariably produces some degree of opacity of the refracting media.

It has been pointed out by Mr Emsley that the methods described in the paper aim at observing the structure and measuring the internal dimensions of the *living* eye.

In reply to Comdr. Langlands, it is important that the slit image be free from aberration. The employment of aspherical lenses, however, does not seem to be necessary to obtain this condition. The object in designing this apparatus was to obtain a useful instrument which was easy of construction and did not necessitate a considerable outlay for special optical parts. Knowing that in the Zeiss instrument a figured lens was used for the purpose of projecting the image of the slit, I thought that difficulty might be experienced in this matter, but found on trial that a good 2-inch microscope objective gave an image in which aberration was not apparent under the magnification used in the microscope, *i.e.* 20 to 30 diameters.

With regard to the suggestion that under certain conditions of illumination the light scattered from the surfaces of the eye may be polarized, I regret that I have made no observations on this subject. In the light of the findings of Koeppe, as mentioned by Mr Gosling, this should prove an interesting field of study.

I should like to thank Mr Smith for the suggestion of using light of the extreme violet end of the spectrum for the purpose of photographing the eye. When time permits of further experiments in this work I hope to apply this method. Up to the present the method has been to use the light of a carbon arc with high speed panchromatic or isochromatic plates, the exposure being about one-fifth of a second. Although radiation in the extreme violet, or slightly into the ultra-violet, produces only little retinal response and therefore does not cause discomfort to the subject at the time, much exposure to this light is injurious to the tissues of the eye. I think it would be unwise to produce obvious fluorescence in the crystalline lens except perhaps for periods of very short duration.

Mr Gosling has referred to the method of measuring the apparent depth of the anterior chamber by means of the Ulbrich drum. As I pointed out in the paper, the measurement is arrived at in this case by determining the amount of movement necessary to focus the microscope successively upon the two surfaces whose separation is being measured. The accuracy of the result is thus dependent upon the observer's ability to estimate exact focus and to control his own accommodation.

Vogt has stated in his works the importance in slit lamp microscopy of maintaining a perfect focus of the slit image upon the part under observation. This presents another difficulty in measuring the anterior chamber by means of the Ulbrich

drum, as it is not only necessary to focus the microscope, but the image of the slit must also be moved from one surface to the other. These adjustments would, except to the very skilled worker, take some time to make. It is important that the reading should be taken as speedily as possible, as the permissible error in the measurement of the depth of the anterior chamber should not exceed 0.1 mm. and the risk of the patient moving by this amount, if the time necessary for the adjustments is lengthy, is considerable. By the method which I have described with the new instrument, I find that the one adjustment necessary can be made within 2 seconds.

Although, for clinical purposes, the use of schematic values for the cornea is sufficiently accurate in computing the real depth from the apparent value, it is desirable as far as possible to determine these values in the actual eye which is being studied. The radius of curvature of the cornea at least can be readily obtained by means of the ophthalmometer.

With regard to the fitting of a monocular microscope instead of a binocular instrument, this was done in the existing model very largely for reasons of economy. As Prof. Cheshire has mentioned, the binocular microscope could be mounted upon this instrument.

Mr Gosling thinks that the angle between the illuminating beam and the microscope could not be made sufficiently acute in this form of mounting to allow of the observation of the deeper parts of the vitreous humour. I find it is possible to reduce this angle to less than 20° , which permits of illumination and observation of structures well behind the crystalline lens.

THE CHOICE OF WAVE-LENGTHS FOR ACHROMATISM IN TELESCOPES

BY LT.-COL. J. W. GIFFORD, F.R.A.S.

MS. received, 5th December, 1923. Read and discussed, 14th February, 1924.

ABSTRACT. In correcting a combination of two glasses for chromatism, focal lengths for two colours of the spectrum are made equal. The outstanding error for other such colours is called the secondary spectrum. This paper has been written for the purpose of determining the effect on the secondary spectrum of varying the two colours chosen. These effects, calculated by trigonometrical trace, are illustrated by graphs and tables.

THE object of the present paper is to determine how the secondary spectrum of a simple two-lens achromatic objective is affected by varying the two rays chosen for achromatization. It is a question of physical optics and has little to do with the form of the combination.

Since the time of Dollond and Fraunhofer, whose earlier work was systematized by Lloyd and Coddington, little has been done in this particular direction with the exception of the work of Mr Dennis Taylor*.

In a paper† read before this Society on 15th April, 1915, the author has already drawn attention to the subject, in reference more especially to the undoubted advantage of using certain wave-lengths, not previously employed, for the correction of apochromatic triple objectives. The same question about doublets was outlined, but in the present paper is more fully discussed.

We have been accustomed to achromatize the object glasses of our telescopes and other instruments by equalizing focal lengths for wave-lengths 6563 (line *C*) and 4861 (line *F*) with wave-length 5893 (the mean *D* line) for the principal ray of shortest focus, this being brought about by associating a positive lens of crown glass with a negative lens of flint glass with suitably proportioned dispersive powers, thus:

Crown	Flint
$p = \frac{\mu_F - \mu_C}{\mu_D - 1}$	$p' = \frac{\mu'_F - \mu'_C}{\mu'_D - 1}$
$f = \frac{p'}{p - p'}$	$f' = \frac{p}{p - p'}$
$\frac{1}{f} + \frac{1}{f'} = \frac{1}{F} = 1,$	

where μ is the refractive index of the line indicated, p the dispersive power of the glass, and f and f' the foci for a combined focus F , of value unity.

* *Monthly Notices, R.A.S.* 54 (1893-4) 67.

† "A Revised System of Comparative Optics," *Trans. Opt. Soc.* 14 (1914-15) 187.

Of late years some doubt has prevailed as to whether *C*, *F*, and *D* are actually the best lines for this purpose. The formula given refers to a combination without actual thickness, which is unthinkable. But the influence of thickness on the achromatism is generally inconsiderable, being only about one-tenth of the total error. It becomes serious only with increasing focus and aperture, as may be gathered from the results for the two doublets given in Table I, both being calculated from identical data, the first without thickness, the other with it.

Table I. *Lens without Thickness*

ϕ	BF_C	BF_A	BF_F	$BF_C - BF_A$	$BF_A - BF_F$	R_0	$R_3 = R_4$	R_5	$s.a.$
$\frac{\mu_F - \mu_C}{\mu_A - 1}$	10.00708	10.00251	10.00929	0.00457	-0.00678	4.147786	-4.075942	∞	-0.002509

Lens with Thickness

$t_1 = 0.125$, $t_2 = 0.000$ (cement), $t_3 = 0.075$.

ϕ	BF_C	BF_A	BF_F	$BF_C - BF_A$	$BF_A - BF_F$	R_0	$R_3 = R_4$	R_5	$s.a.$
$\frac{\mu_F - \mu_C}{\mu_A - 1}$	9.85786	9.85289	9.85904	0.00498	-0.00617	4.147786	-4.033861	∞	-0.002251

ϕ = dispersive power; BF = back focal length, the distance from the last surface along the axis at which the emergent ray intersects the axis; $s.a.$ = outstanding spherical aberration; t = thickness.

The two glasses used in this investigation were Messrs Chance Bros. No. 4497 melting of ordinary hard crown and No. 4380 of ordinary heavy flint. All the doublets were computed for an equivalent focal length of 10 inches and an effective aperture of 1 inch, the ratio of focal length to aperture thus being 10. With the exception of the one without thickness, the central thickness (along the axis) of each positive element was 0.125 inch, that of each negative element 0.075 inch.

These two elements were cemented together, so that the thickness between them need not be considered. In all, nine such doublets with thickness have been computed by rigorous trigonometrical ray-tracing, three where the shortest focal length was that of *A*, three where the shortest was that of *D*, and three where it was that of *Hg*, the well-known mercury line of wave-length 5461. Each of these three groups consisted of three separate complete calculations, one where *C* and *F* were equalized, one for *D* and *E*, and one for *D* and *F*. The back focal lengths for the five lines *C*, *D*, *A*, *E*, and *F* are given in Table II.

Table II. *Values of Back Focal Length for Paraxial Ray (P.R.) and $\frac{3}{4}$ Edge Ray (E.R.)*

	Group I (Λ)					
	$\phi = \frac{\mu_F - \mu_C}{\mu_A - 1}$		$\frac{\mu_E - \mu_D}{\mu_A - 1}$		$\frac{\mu_F - \mu_D}{\mu_A - 1}$	
	P.R.	E.R.	P.R.	E.R.	P.R.	E.R.
<i>C</i>	9.85147	9.85057	9.85248	9.85107	9.85951	9.86164
<i>D</i>	9.84689	9.84641	9.84763	9.84677	9.85335	9.85342
Λ	9.84560	9.84555	9.84618	9.84565	9.85120	9.85158
<i>Hg</i>	9.84608	9.84623	9.84656	9.84623	9.85114	9.85172
<i>E</i>	9.84691	9.84694	9.84724	9.84723	9.85119	9.85208
<i>F</i>	9.85044	9.85371	9.85040	9.85106	9.85270	9.85430

Group II (D)

	$p = \frac{\mu_F - \mu_D}{\mu_D - 1}$		$\frac{\mu_E - \mu_D}{\mu_D - 1}$		$\frac{\mu_F - \mu_D}{\mu_D - 1}$	
	P.R.	E.R.	P.R.	E.R.	P.R.	E.R.
C	9.85029	9.84914	9.85115	9.84979	9.85743	9.85578
D	9.84573	9.84547	9.84546	9.84546	9.85131	9.85141
A	9.84444	9.84455	9.84487	9.84431	9.84916	9.84959
Hg	9.84492	9.84528	9.84523	9.84492	9.84910	9.84970
E	9.84574	9.84580	9.84592	9.84586	9.84915	9.85000
F	9.84927	9.85053	9.84907	9.84978	9.85067	9.85106

Group III (Hg)

	$p = \frac{\mu_F - \mu_D}{\mu_{Hg} - 1}$		$\frac{\mu_E - \mu_D}{\mu_{Hg} - 1}$		$\frac{\mu_F - \mu_D}{\mu_{Hg} - 1}$	
	P.R.	E.R.	P.R.	E.R.	P.R.	E.R.
C	9.85093	9.85103	9.85204	9.85067	9.85946	9.85902
D	9.84635	9.84602	9.84719	9.84635	9.85335	9.85346
A	9.84506	9.84522	9.84576	9.84521	9.85120	9.85158
Hg	9.84555	9.84573	9.84580	9.84580	9.85114	9.85173
E	9.84647	9.84643	9.84680	9.84679	9.85119	9.85209
F	9.84990	9.85124	9.84996	9.85064	9.85277	9.85430

As far back as 1904 the author noticed that a line in the spectrum of lead of wave-length 5607 (since corrected to 5609) was so situated, with regard to himself and many others who used it, that it was just at the point of maximum intensity for visual perception, more so than either the mean *D* line or the mercury line already referred to. It was called by the Greek letter Λ . In the following year my friend, Mr E. M. Nelson, recommended the Royal Microscopical Society to adopt it as the standard wave-length for white light in their Numerical Aperture Table. It was at once adopted and has now for many years been in general use by the Society*.

This is a physiological question. As explained by Mr Dennis Taylor, to whom reference has already been made, the eye chooses the point of maximum visual effect arbitrarily and it is even now by no means claimed that the maximum for everyone is Λ . There are probably people for whom even line *E* is the maximum, but the claim is made that the average eye, of the Anglo-Saxon race at least, chooses approximately line Λ .

For this reason this line would be advantageously chosen for reference rather than the old mean *D* line, or the mercury line just referred to and until now used so generally on account of its easy accessibility from the mercury arc lamp.

The doublet object glasses were corrected for spherical aberration, using the paraxial ray and a ray incident at 0.375 inch from the axis† for this purpose, for lines Λ , *D*, and Hg respectively, in the three corresponding groups. The results are given in Table III, from which it will be seen that the outstanding spherical aberration in no case amounts to as much as one-thousandth of an inch. The maximum was -0.00084 and the average ± 0.00036 inch.

* *Journ. Roy. Micr. Soc.* (Feb. 1905) 107.

† A ray actually incident at the edge was not traced, but one at the representative height of 0.375 inch, or three-quarters of the distance from centre to edge. The surfaces on either side of a corresponding zone have equal areas.

Table III. *Radii, etc.* $i_1 = 0.125$, $i_2 = 0.000$ (cement), $i_3 = 0.075$.Group I (Λ)

p	R_0	$R_2 = R_4$	R_6	<i>s.d.</i>
$\frac{\mu_F - \mu_C}{\mu_\Lambda - 1}$	4.030779	-4.151051	140.6814	0.000052
$\frac{\mu_E - \mu_D}{\mu_\Lambda - 1}$	4.045947	-4.168240	154.1768	0.000534
$\frac{\mu_F - \mu_D}{\mu_\Lambda - 1}$	4.173859	-4.186899	4096.472	-0.000383

Group II (D)

p	R_0	$R_2 = R_4$	R_6	<i>s.d.</i>
$\frac{\mu_F - \mu_C}{\mu_D - 1}$	4.030779	-4.150162	141.2373	0.000250
$\frac{\mu_E - \mu_D}{\mu_D - 1}$	4.045947	-4.167215	154.9376	0.000838
$\frac{\mu_F - \mu_D}{\mu_D - 1}$	4.173859	-4.185262	5131.961	0.000095

Group III (Hg)

p	R_0	$R_2 = R_4$	R_6	<i>s.d.</i>
$\frac{\mu_F - \mu_C}{\mu_{Hg} - 1}$	4.030779	-4.150518	140.9610	-0.000186
$\frac{\mu_E - \mu_D}{\mu_{Hg} - 1}$	4.045947	-4.167795	154.4523	0.000325
$\frac{\mu_F - \mu_D}{\mu_{Hg} - 1}$	4.173859	-4.186831	4107.380	-0.000564

In judging the respective values of the differences of their back focal lengths from those for Λ it is necessary to take into account the photometric values of the different wave-lengths used. Thus it will be seen that light for some distance on both sides of, and including, lines C and F has small value when compared with similar light in the regions of D and E and that these again are surpassed by that at Λ . The graphs shown in Figs. 1 and 2 have been drawn with focal lengths and oscillation frequencies (reciprocals of wave-lengths) as abscissae and ordinates respectively. It will be at once apparent that, however important the proper choice of the visual maximum for shortest focal length may be in non-achromatized combinations, it makes little difference here, for in the result the shortest is always that for Λ . But this is by no means the case with the two rays for which focal lengths

are equalized. When $p = \frac{\mu_F - \mu_D}{\mu - 1}$, we have a raising of the value of the spectrum extending from Λ to F and beyond, and a corresponding depreciation of that from Λ to C . Telescope doublets which have been constructed from such data have given an effect on a landscape approaching that of certain painters who exalt the values of their yellows, blues, greens, and violets at the expense of their oranges and reds, in order to enhance the attractiveness of otherwise uninviting spots.

When $p = \frac{\mu_F - \mu_C}{\mu - 1}$ very little of this effect remains in my case, and none at all when $p = \frac{\mu_E - \mu_D}{\mu - 1}$. I have been pressed to give my personal opinion and without

hesitation give it for $\frac{\mu_E - \mu_D}{\mu - 1}$. Although F and C give almost as well balanced a graph, yet it is not quite so, for, in traces for the three-quarters edge ray, in every case the resulting focal length for F is greater than that for C . Under the same conditions, when E and D are used in the formula, the resulting focal lengths for C and F are practically equal throughout. In this question the paraxial ray has no importance, for it exists in theory only. Thus, to get C and F equal, the calculation must be made with E and D . I always now compute my doublets in this way and invariably find them the best when used with a really achromatic eyepiece. When this is not the case, good results may even be obtained with F and D , for instance, with an eyepiece much over-corrected for colour. In all such cases computers must decide for themselves. Huygenian and other non-achromatic eyepieces should always be computed with μ_A .

The graphs (Figs. 1 and 2) correspond with every focal length in the groups. It is noteworthy that, whatever p may be, the focal lengths for Λ are always the shortest.

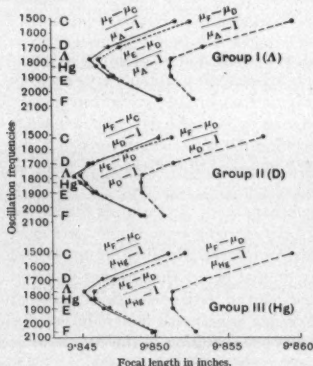


Fig. 1. Paraxial Ray.

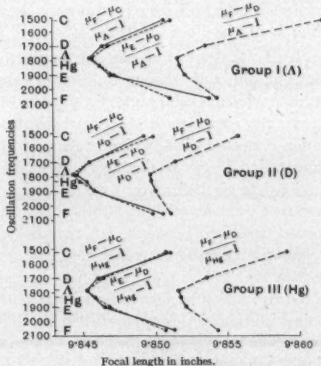


Fig. 2. $\frac{1}{4}$ Edge Ray.

DISCUSSION

Prof. Cheshire: It is fortunate that Col. Gifford has not to content himself with "paper" systems. His computations generally are simply a preliminary to actual construction, and in every case in which I have had an opportunity of seeing these constructions, the theoretically deduced performance has been realized—a high tribute to the capacity and care of the computer. I hope that the Society will receive a number of such contributions from Col. Gifford.

Commander T. Y. Baker: Col. Gifford's proposal to adopt a new pair of wave-lengths for colour correction in place of the customary *C* and *F* raises a certain amount of doubt in my mind respecting the practicability of his scheme. With two lines as close together as those he advocates the reciprocal dispersion power will be changed from

$$(\mu_D - 1)/(\mu_F - \mu_C) \text{ to } (\mu_A - 1)/(\mu_E - \mu_D).$$

The denominator of the fraction is thus from one-third to one-half of its previous value and the ν of the glassmaker's catalogue will become two or three times as great as before. The accuracy of the figures in the catalogue depends on the accuracy with which the refractive indices for the standard wave-lengths are measured, so that Col. Gifford's proposals, if adopted, would seem to involve the glassmaker in a more careful measurement of his refractive indices if he wishes his catalogue specifications to be accurate.

Dr J. S. Anderson: I think that, from many points of view, the present standard wave-lengths are far from ideal. I am surprised to see that the author retains the *D* line as a standard. This, being a doublet, is not a suitable line for routine measurements with the Pulfrich refractometer. It is possible that one of the helium or neon lines would be more satisfactory.

Mr J. W. Perry: Col. Gifford has given us, as the outcome of considerable labour and thought, a discussion of no mean value bearing upon the secondary spectrum in its various possible and favourable manifestations in subjective instruments. Under the circumstances upon which the discussion is based the chromatic aberration concerned is sufficiently defined by stating the wave-length for which the intersection-length attains an extreme value. The practical realization of this correction has given further occasion to the vexed question, regarding the choice of standard wave-lengths, dwelt upon by foregoing speakers. In deciding the fundamental question, however, viz. the choice of the extreme wave-length, one must necessarily be guided by data concerning relative spectral visibility for an average human eye. This is not a stationary property, in that before the range of modified chromatic vision at low intensities is reached marked variation with intensity of retinal illumination is apparent. It would therefore appear that it were in practice unnecessary for general purposes to aim at extreme precision in the definition of simple achromatism.

Mr T. Smith: The author has carried out a laborious piece of work which should prove distinctly useful to those who wish for precise information on the best type of chromatic correction. It is a question which has to be determined by ocular experience with instruments constructed to accurately known data from glasses of measured indices and dispersions. It would be advantageous if all workers who have made experiments under such conditions would place them on record for general use, so that the extent to which a consensus of opinion favoured a particular type of correction could be known.

Col. Gifford: I am very grateful that Prof. Cheshire has drawn attention to the fact that most of my computations latterly proceeded to actual construction. This

paper originated in deductions drawn from observations on an artificial star and is an attempt to justify them with figures.

I quite see Commander Baker's point. It would perhaps involve the glassmaker in more careful measurements, but the lines *E* and Λ are excellently sharp and clear and very much easier to measure with accuracy than *C* or *F*. Of course *D* would remain the same, but the glassmakers take more trouble with this line than with any other already, and as it is a double line of which the components are wide enough apart for the pointer to be set on the dark line separating them, this line is fairly sure of accuracy. But an interpolation table has been prepared from an exhaustive list of refractive indices, accurate within an error of from one to three units in the fifth decimal place, prepared by the author at the request of the late Sir David Gill and published in a paper in the *Proceedings of the Royal Society**. In the interpolation table referred to refractive indices for twelve wave-lengths are placed in contiguous columns in order of their mean dispersions, and the new glass being allocated to its position between any two columns by observing its refractive indices for *C*, *D*, and *F* only, a simple proportion sum enables those for any other wave-length and certainly for those of *E* and Λ to be found with much accuracy. This, I think, should meet Commander Baker's difficulty.

I agree with Dr Anderson's view. The present lines are most unsatisfactory, but I have no intention of retaining the *D* line in use. It is only used in the paper for the purpose of comparison. I would substitute Λ for it on every occasion. I am unable to think of any line in the spectrum of helium in a similar position to Λ , but there may be in that of neon. Common lead, which produces Λ , serves the purpose admirably.

I am quite ready to admit Mr Perry's argument. I have already said that "there are probably people for whom line *E* is a maximum"; possibly I should have added "and times occasionally when it is a maximum for everyone."

I quite agree with Mr Smith's remarks. A record of the corrections used for every lens which performed particularly well under test after construction would be invaluable, could the makers be induced to take the trouble.

* *Proc. Roy. Soc.* 87 (1912) 189.

A REFERENCE SYSTEM FOR PRIMARY ABERRATIONS

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ABSTRACT. All the primary aberrations are fully known when the state of correction for all points on the axis is given. The condition in which the whole length of the axis is free from aberrations is suggested as a standard of reference for general investigations, the departures from which in the cases normally required are functions of the positions of the object, the stop, and the focal length only. The inevitable departures from the standard in the case of a thin lens and of a system of two separated thin lenses for first order aberrations are considered and shown to affect the curvature terms. In general the distinctions between the new reference system and that in which an inverted image equal to the object and a stop in a unit surface are adopted, are confined to the symmetrical aberration terms.

It is well known that in an optical system which is corrected for an object occupying one plane there is failure to satisfy the conditions for correction in another plane by a definite amount. A familiar example of this inconsistency is afforded by the two well-known sine conditions of Herschel and of Abbe. The former states that if a ray starts from a point on the axis in a medium of refractive index μ making an angle ψ with the axis, and meets the axis at an angle ψ' in the image space where the refractive index is μ' , and if the system is without aberration for all rays passing through this axial object point, where the transverse magnification is G , then a neighbouring axial point will be without aberration if, and only if,

$$\mu \sin \frac{1}{2}\psi = G\mu' \sin \frac{1}{2}\psi' \quad \dots\dots(1).$$

On the other hand, the Abbe sine condition states that under similar circumstances a neighbouring point of the transverse object plane will only be imaged without aberration if the condition

$$\mu \sin \psi = G\mu' \sin \psi' \quad \dots\dots(2),$$

is satisfied. In general these conditions are inconsistent with one another, and they serve to show that, whatever the order of the aberration may be, the conditions for freedom from primary aberrations are necessarily functions of the position of the object.

In an earlier investigation* these changes in the conditions for different object positions have been investigated in detail; it was shown that the position of the stop must be specified as well as that of the object, and that it then becomes possible

* "The Changes in Aberrations when the Object and Stop are moved" *Trans. Opt. Soc.* 23 (1921-22) 311.

to express the aberrations for any given object and stop positions in terms of those for any other positions. Accordingly it is only necessary to investigate the aberrations for a single object position and a single stop position to derive the full properties of the system. For the systematic study of lenses it is desirable to adopt definite positions for reference purposes, so that a direct comparison of one system with another may be made without any additional calculations. The choice made is not of the first importance, though definite advantages are gained by paying attention to symmetry. One symmetrical system which has been employed makes use of the two principal foci as the axial points of the image and stop surfaces respectively. It is found that a more convenient choice is the surface of magnification $+1$ for the stop and that of magnification -1 for the image. In this case the transposition of object space and image space involves nothing more than changes of sign—a manifest advantage. It is here intended to propose a third alternative which has all the advantages of the previous choice as regards symmetry and in addition other advantages which are only likely to be fully appreciated when some developments of lens theory which cannot be discussed in this paper are taken into consideration.

Before entering on a discussion of this proposal it is important to note that it is exceptional for the conditions for the correction of aberrations to depend on the position of the object. In the earlier paper to which reference has already been made the aberrations were divided into series, a classification which cuts across the familiar division into orders. It is only the aberrations of the series which has been named the zero series which exhibit this peculiarity of irreconcilable vanishing conditions at different magnifications. These are precisely the aberrations which appear in an axial plane of a lens system, and are generally known as the primary aberrations. In all other cases, if the conditions are satisfied for one object position they are satisfied for all positions. It is therefore only necessary to consider these primary aberrations in discussing reference systems for aberrations.

The primary aberrations are fully determined by a knowledge of the paths of all rays in a plane through the axis of the system. All these rays intersect the axis, and a knowledge of the axial intersection point obviously furnishes one of the two co-ordinates required to define the position of any ray. It is less obvious that the remaining co-ordinate of the ray, its direction, is also known if the intersection points in general are known. That this is the case is illustrated in the absence of axial aberration by Herschel's condition (1). In general the direction of the ray is given by the cosine law*. The specification of axial aberrations alone is thus sufficient for the determination of all primary aberrations, and is the reference system it is proposed to employ for recording the state of correction of optical systems generally. One of its obvious advantages is that it does not involve an arbitrary selection of object and stop positions. The expressions for the addition of aberrations are also invariable for the reason that no particular object position is considered. Incidentally the coefficients may be assimilated by combination to any system of co-ordinates or ray parameters.

* "The Optical Cosine Law," *Trans. Opt. Soc.* 24 (1922-23) 31.

Taking the normal case in which the system is in air, when there is no aberration on the axis and all the aberrations except those of the zero series vanish, the Eikonal referred to the principal foci as reference points is $\mathcal{E}$, where

$$\begin{aligned}\mathcal{E}\kappa &= -2 \sin \psi \sin \psi' (1 + \cos \psi)^{-\frac{1}{2}} (1 + \cos \psi')^{-\frac{1}{2}} \\ &= -2 (1 - \cos \psi)^{\frac{1}{2}} (1 - \cos \psi')^{\frac{1}{2}} \quad \dots(3)^*.\end{aligned}$$

This expression must be expanded in terms of $\sin \psi$ and $\sin \psi'$ to yield the relations between the transverse and the axial aberrations. Since

$$\{2(1 - \cos \psi)\}^{\frac{1}{2}} = (1 + \sin \psi)^{\frac{1}{2}} - (1 - \sin \psi)^{\frac{1}{2}} = \sum \frac{4^p!}{2^{4p}(2p+1)!2p!} \sin^{2p+1} \psi,$$

it follows from (3) that the terms in $\mathcal{E}\kappa$ which do not represent axial aberrations are

$$- \left[\sum \frac{4^p!}{2^{4p}(2p+1)!2p!} \sin^{2p+1} \psi \right] \left[\sum \frac{4^p!}{2^{4p}(2p+1)!2p!} \sin^{2p+1} \psi' \right].$$

The leading terms are

$$\begin{aligned}& -\sin \psi \sin \psi' \left[1 + \frac{1}{8} (\sin^2 \psi + \sin^2 \psi') + \frac{1}{128} (7 \sin^4 \psi + 2 \sin^2 \psi \sin^2 \psi' + 7 \sin^4 \psi') \right. \\ & \quad + \frac{1}{1024} (33 \sin^6 \psi + 7 \sin^4 \psi \sin^2 \psi' + 7 \sin^2 \psi \sin^4 \psi' + 33 \sin^6 \psi') \\ & \quad + \frac{1}{32768} (715 \sin^8 \psi + 132 \sin^6 \psi \sin^2 \psi' + 98 \sin^4 \psi \sin^4 \psi' \\ & \quad \quad \quad + 132 \sin^2 \psi \sin^6 \psi' + 715 \sin^8 \psi') \\ & \quad \left. + \dots \right]\end{aligned}$$

These may be written in the form

$$\begin{aligned}\mathcal{E}\kappa &= -(2) \left[1 + \frac{1}{8} \{(1) + (3)\} + \frac{1}{128} \{7(1)^2 + \frac{1}{2}(2)^2 + \frac{7}{8}(1)(3) + 7(3)^2\} \right. \\ & \quad + \frac{1}{1024} \{(1) + (3)\} \{33(1)^2 + 4(2)^2 - 30(1)(3) + 33(3)^2\} \\ & \quad + \frac{1}{32768} \{715(1)^4 + 44(1)^2(3) + 88(1)^2(2)^2 + \frac{7}{8}(1)^2(3)^2 \\ & \quad \quad \quad + \frac{27}{8}(1)(2)^2(3) + \frac{1}{16}(2)^4 + 88(2)^2(3)^2 + 44(1)(3)^3 + 715(3)^4\} \\ & \quad \left. + \dots \right] \quad \dots(4),\end{aligned}$$

where

$$(1) = M^2 + N^2, \quad (2) = MM' + NN', \quad (3) = M'^2 + N'^2,$$

as in the earlier paper. The division of the terms involving both ψ and ψ' to the third or higher orders is that which introduces no terms of any but the zero series. The expression (4) may be compared with equation (13) of the previous paper which gives the value of $\mathcal{E}$ when aberrations for the transverse magnification G are corrected, the stop magnification being S . Equation (4) may be expressed in terms of the variables appropriate for the object magnification G and the stop magnification S by means of the substitution

$$\begin{aligned}(1) &= S^2 I - 2SGII + G^2 III, \\ (2) &= SI - (S + G)II + GIII, \\ (3) &= I - 2II + III.\end{aligned}$$

The results suggest the use of quantities for the expression of primary aberrations which are closely related to the Δ 's introduced in another connection†. In addition it may be remarked that the symmetry of the expression for $\mathcal{E}$ suggests that a

* *Trans. Opt. Soc.* 24 (1922-23) 38.

† *Trans. Opt. Soc.* 24 (1922-23) 176, footnote.

system of co-ordinates based on the values $+1$ and -1 for S and G is likely to lead to simpler expressions than when other selections are made. This may be inferred otherwise, since the two sine conditions become consistent at these points, and are also consistent with the tangent condition for freedom from distortion. It may be anticipated that the distinctions between the two reference systems will not affect the unsymmetrical aberrations.

To illustrate the system of reference, the first order aberrations for combinations of thin lenses may be considered. If there are more than two separated lenses all these aberrations may be given arbitrary values simultaneously, and the absence of spherical aberration along the whole axis is possible. For a single lens the expression for the spherical aberration is a numerical multiple of

$$(\gamma - 4\beta\mathcal{S} + 6a\mathcal{S}^2)(1 - G)^4\kappa^{-1},$$

where κ is the power of the lens, G is the magnification, $a = \frac{1}{2} + \frac{1}{3}\varpi$, ϖ being the Petzval coefficient, β is the sum of the curvatures of the unit surfaces and can be given any desired value by a change in the form of the lens, γ is the spherical aberration coefficient for magnification -1 and depends upon the constitution of the lens as well as on its form, and

$$\mathcal{S} = \frac{1 + G}{1 - G}.$$

The conditions for entire freedom from first order axial aberration are evidently

$$\gamma = \beta = a = 0.$$

Since β and γ are under control the first two conditions can readily be satisfied. The third condition cannot be met since ϖ is positive, and in fact almost invariable in value. Since the curvature of the primary focal surface is $(3 + \varpi)\kappa$, it will be observed that the discrepancy between the axial conditions and those for $G = -1$, $S = 1$ only affects the curvature term, one of the symmetrical aberrations.

The other case of particular interest is that of two separated thin lenses. By the choice of suitable values for the β and γ of each lens four conditions can be satisfied. Since there are five primary aberrations and the same number of conditions for the absence of spherical aberration at all magnifications, a further condition remains to be satisfied for the complete absence of axial aberration. From the general considerations that have been mentioned this condition would be expected to be

$$\kappa_1\alpha_1 + \kappa_2\alpha_2 = 0$$

and this may be verified as follows.

From the addition law for spherical aberration the amount of the aberration in a system of two separated thin lenses is a multiple of

$$\begin{aligned} &(\gamma_1 - 4\beta_1\mathcal{S}_1 + 6a_1\mathcal{S}_1^2)(g - G)^4\kappa_1^{-1} \\ &+ (\gamma_2 - 4\beta_2\mathcal{S}_2 + 6a_2\mathcal{S}_2^2)(1 - g)^4\kappa_2^{-1}, \end{aligned}$$

where G is the magnification for the complete system, g for the second lens, and

$$\mathcal{S}_1 = \frac{g + G}{g - G}, \quad \mathcal{S}_2 = \frac{1 + g}{1 - g}.$$

In the case of a single lens, two of the four axial points at which the spherical aberration vanishes coincide with the vertex of the lens. The aberration therefore has the same sign immediately on either side of the vertex. A second lens which is to neutralize this aberration must produce aberration which vanishes at the vertex of the first lens and has the same sign on either side. In other words, each component must be corrected for two coincident points at each lens vertex. Let the corresponding values of $\mathcal{S}_1$ be ∞ and $\mathcal{S}_1$, and of $\mathcal{S}_2$ be $\mathcal{S}_2$ and ∞ . Then the aberration is measured by

$$6a_1(\mathcal{S}_1 - \mathcal{S}_1)^2(g - G)^4\kappa_1^{-1} + 6a_2(\mathcal{S}_2 - \mathcal{S}_2)^2(1 - g)^4\kappa_2^{-1}.$$

Now if t is the separation of the component lenses and κ the power of the combination, and

$$\mathcal{S}_1 = \frac{1 + s_1}{1 - s_1}, \quad \mathcal{S}_2 = \frac{1 + s_2}{1 - s_2},$$

the standard magnification relations give

$$g\kappa = \kappa_1 + G\kappa_2$$

$$\kappa = \kappa_1 + s_1\kappa_2$$

$$s_2\kappa = \kappa_1 + s_2\kappa_2$$

so that

$$\mathcal{S}_1 - \mathcal{S}_1 = \frac{2(1 - g)}{t\kappa_2(g - G)} \quad \text{and} \quad \mathcal{S}_2 - \mathcal{S}_2 = \frac{-2(g - G)}{t\kappa_1(1 - g)}.$$

On substituting these values in the expression for the aberration it becomes

$$24(1 - g)^2(g - G)^2(a_1\kappa_1 + a_2\kappa_2)(t\kappa_1\kappa_2)^{-2},$$

showing that a necessary condition for freedom from axial aberration is

$$a_1\kappa_1 + a_2\kappa_2 = 0.$$

This condition evidently permits little departure from equal and opposite powers for the two components. It will be noted from its form that it applies to telescopes as well as to systems of finite focal length.

THE CIRCULAR DIVIDING ENGINE OF EDWARD TROUGHTON, 1793

By DAVID BAXANDALL, A.R.C.S., F.R.A.S.

Exhibited, 14th February, 1924.

THE history of optics, as of astronomy, navigation, and surveying, is closely interwoven with the history of the accurate division of the circle.

Last year Messrs Cooke, Troughton & Simms lent this circular dividing engine for exhibition in the Science Museum, and as it occupies an important position in the history of machine dividing, it was thought that Members of the Society might be glad to have the opportunity of examining the mechanism and seeing it in action.

I shall not attempt to describe in detail the construction of this dividing engine, but I propose simply to give a few dates and particulars referring to the history of the machine.

Henry Hindley of York, about 1740, was the first to construct an engine for cutting the teeth in clock wheels and for dividing instruments. In this he used the roller method for the original division of the dividing plate, which was actuated by an endless screw. A full account of his work was given by Smeaton in 1786^{(1)*}.

In 1768 the Duc de Chaulnes described in detail a circular dividing engine⁽²⁾ in which, however, the wheel was not turned by an endless screw; the wheel and the work screwed to it were actuated by a clamp and screw for slow motion, bringing a division of the limb under the wire of a fixed microscope, the division being then cut by a point and frame.

Two years before this Jesse Ramsden had made his first dividing engine, with a dividing plate 30 inches in diameter, but though this engine gave more accurate results than by the ordinary dividing plate method, and was good enough for dividing the circles of the common surveying instruments, it was not sufficiently accurate for nautical instruments used in the determination of position.

In 1775 Ramsden had completed his second and very much better engine. A sextant divided by it was examined by John Bird, the leading hand-divider at that time, who reported favourably on it. For his invention Ramsden received £615 from the Commissioners of Longitude on condition that he would divide sextants and octants for the trade at the rate of 6s. per sextant and 3s. per octant, also that he should instruct a certain number of persons (not exceeding ten) in the method of making and using the engine, during the period 28th October 1775 to 28th October 1777, the engine to become the property of the Commissioners.

Ramsden's engine⁽³⁾ consists of a horizontal wheel or plate 45 inches in diameter, which turns on a vertical axis; its outer edge is ratched or cut into 2160 teeth, into which an endless screw gears. The downward stroke of a treadle turns the screw

* The small numbers in the text refer to the list of references at the end.

through any portion of a revolution as fixed by the setting of suitable mechanism. By means of a free wheel on the worm axis, the upward stroke of the treadle leaves the worm stationary. The circle to be divided is centered and fixed securely to the horizontal plate of the engine, and after each downward stroke of the treadle a division is cut by hand, the cutting point being carried in a frame (invented by Hindley) which allows only a radial to-and-fro motion of the point. One forward revolution of the screw advances the wheel through 10 minutes of arc. A brass plate on the screw arbor is divided into 60 parts, so that one division of this corresponds to 10 seconds of arc on the wheel. This engine was used continuously by Ramsden until his death in 1800. Since that time it has been in the possession of several dividers, and is now in the U.S. National Museum at Washington(7).

In 1778 John Troughton completed an engine which had occupied him for three years. It was in general construction like that of Ramsden, but according to Edward Troughton (John's younger brother) it was thought to be superior in point of accuracy. Writing in 1830, Edward Troughton states(4):

The excellent engine of my late brother being fully four feet in diameter gave the operator, when at work near the centre, a position so painful, that it had done no good to either his health or my own, and had materially injured that of a worthy young man then my assistant; it was evident that, by making one of smaller dimensions, this evil would in a great measure be removed, and I foresaw that by employing my own method of original dividing from which to rack the plate, a considerable reduction might be effected without any sacrifice to accuracy. I also perceived, that by contriving the parts with more simplicity than Ramsden had done, I could get through the work at less than two-thirds of the labour and expence. Such were my motives for making an engine and the work was accomplished in the year 1793.

The description was first given in 1830 by Troughton himself at Brewster's request. The engine was driven by a treadle (now missing) and the divisions were cut by hand, as in Ramsden's engine. The circle is 34 inches in diameter, the worm has 20 threads to the inch, and the edge of the plate is ratched into 2160 teeth.

For ratching the plate, John Troughton, like Ramsden, had made notches in the threads of the worm parallel to the axis, and considerable inconvenience had been experienced through the whole length of each cut coming into action at once, causing a jerking motion. Edward Troughton got rid of this by making the notches in spirals, which give the unforeseen advantage that no duplicate driving screw was found to be necessary, though one had been prepared. This original spirally notched worm or "hob" is seen in Figs. 1 and 2.

The engine plate was cast in one piece of brass except the circular rim, which was formed, without soldering, of one piece of fine plate brass; it is rabbeted upon the extremities of the radii.

The original division of the plate is by Edward Troughton's method(5), first described by him in 1809. In a letter to the Astronomer Royal which accompanied the paper Troughton says:

How a young artist, who may be just beginning to make his way to fame or wealth, may receive it, I know not; but I wish him to understand that I consider myself now in the act of making him a very valuable present.

In the first part of the paper he says:

Ramsden's well-known method of dividing by the engine unites so much accuracy and facility, that a better can hardly be wished for; and I may venture to say that it will never be superseded, in the division of instruments of *moderate radii*. It was well suited to the time in which it appeared; a time when the improvements made in nautical

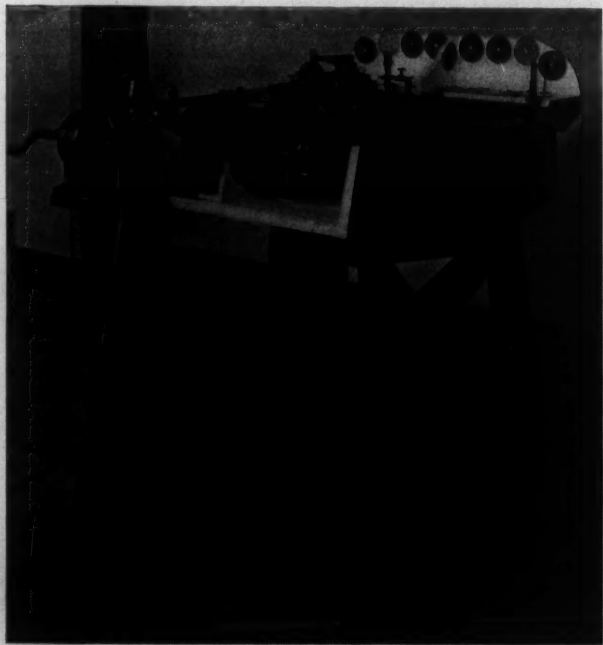


Fig. 1. General View.

astronomy, and the growing commerce of our country, called for a number of reflecting instruments, which never could be supplied, had it been necessary to have divided them by hand: however, as it only applies to small instruments, it hardly comes within the subject of this paper.

He then describes in detail his own method of original division "as practised on a late occasion, in the dividing of a four feet meridian circle, now the property

of Stephen Groombridge, esq. of Blackheath." This original instrument is in the Science Museum.

Troughton states that in his dividing engine and those of Ramsden's construction the operator can cut about 24 divisions per minute and can continue at this rate for hours, allowing for slight interruptions. In a single minute as many as 30 divisions can be cut, but this rate cannot be maintained. There were in London in 1830 some ten or twelve circular dividing engines, mostly copies of Ramsden's second engine.

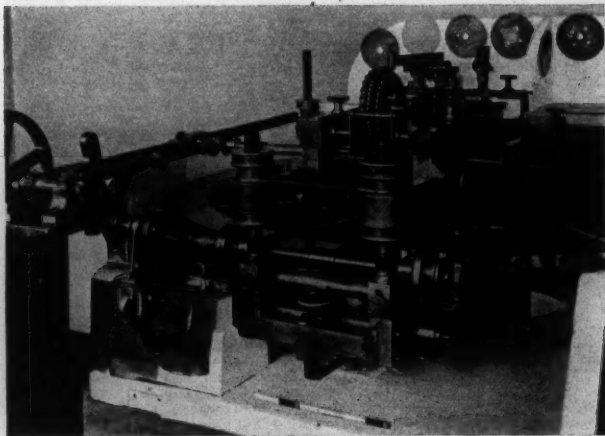


Fig. 2. Showing details of worm drive and tracelet mechanism.

In 1826 William Simms went into partnership with Troughton, who retired from business in 1831 and died in 1835. In 1843 William Simms had completed his own dividing engine (6), which he described before the Royal Astronomical Society in 1843. The plate was 46 inches in diameter and was divided with extreme care on a ring of silver into 4320 divisions, adopting Troughton's method with some slight modifications. A single cutter, mounted in the endless screw frame, was used for ratching the edge of the plate, and as each of the 4320 divisions in order "was brought to coincide with the wires of a powerful microscope, the cutter was entered, and three circulations of the engine plate completed the work." Mr Simms, in his paper, says "I was not without hope that the teeth upon the edge would by this means be cut as truly as the original divisions themselves, and this expectation has, I believe, been fully realised."

A new and important feature in this engine was the mechanism by which the engine became self-acting or automatic. When first used, it was driven by means of a descending weight in an open court adjoining the room in which the engine was placed; later on, it was driven from an overhead shaft by a belt in the usual way.

Fig. 1 shows the engine as exhibited. Some of the details are shown in Figs. 2 and 3. It will be seen that it is something more than the original Troughton engine of 1793. It is really that engine, with self-acting mechanism added by William Simms, probably not long after the larger engine of William Simms had been made. The wooden stand, the worm mechanism, and the large circle were made by Troughton in 1793. The casting (Fig. 1) seen on the left, with its wheelwork, the



Fig. 3. Showing tracelet mechanism, cutting point, and micrometer microscope for adjusting and examining the circle which is being divided.

tracelet frame and cam mechanism for moving the cutting knife up and down, and for regulating the length of cut, are due to William Simms. (For a recent account of the action of some of these parts, see Ref. (8).)

The engine has been in use almost continuously from 1793 until a year or two ago, though of course it has been superseded for more accurate division by the later and better engines constructed by the owners at different periods. During the latter part of its existence it has not been used for graduating instruments reading to a greater accuracy than one minute of arc.

I have to thank Messrs Cooke, Troughton & Simms, Ltd. and Col. H. G. Lyons for allowing the engine to be exhibited, and Mr Mitchell of Messrs Cooke, Troughton & Simms, Ltd. for coming to set up the engine and to show it in operation. My thanks are also due to Col. Lyons for permission to use the photographs to illustrate the paper.

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REPLICAS OF TWO GALILEO TELESCOPES

By DAVID BAXANDALL, A.R.C.S., F.R.A.S.

Exhibited, 10th January, 1924.

REPLICAS of the two Galileo telescopes at Florence were exhibited by the kind permission of Col. H. G. Lyons, F.R.S., Director of the Science Museum. These interesting copies, which have been added recently to the Collections in the Museum, were made in Florence by Cipriani through the kindness of Prof. A. Garbasso, Director of the Museo di Fisica.

At the suggestion of Prof. Geo. E. Hale, made in Florence in May last year, both of the original telescopes, as well as the famous broken object-glass of Galileo, have been tested by Prof. G. Abetti^{(2)*} and Prof. V. Ronchi^(3,4,5). The former

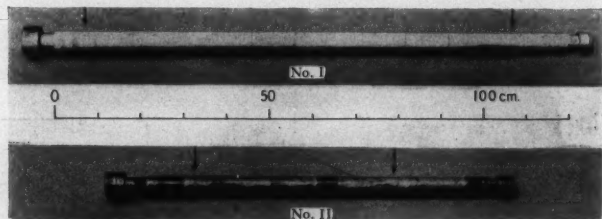


Fig. 1. Replicas of the two Galileo telescopes. The arrows indicate the positions of the inner ends of the draw tubes.

attached the instruments to the Amici equatorial at Arcetri, and tested them by observations of sunspots, lunar craters, Venus, Jupiter, Saturn, and certain double stars. Prof. Ronchi made measurements and tests in the laboratory.

The replicas are shown in Fig. 1, and Fig. 2 shows the internal construction. In No. I, the tube and draw-tubes are made of wood, and in No. II of paper. No. I is covered with paper of a dark maroon colour, while No. II is covered with dark red leather, stamped with a design in gilt. Each lens is held in position by a piece of iron wire, bent into a circle and sprung into the enlarged end of the draw-tube, pressing a paper or cardboard stop against the outer surface of the lens. There is also a wooden stop at the inner end of the O.G. draw-tube in No. I.

The originals are shown in Fig. 2 of Ref. (8), with the broken object-glass, and I regret to say that I gave some particulars of the lenses, which I have since found are incomplete and in one case inaccurate⁽⁶⁾.

* The small numbers in the text refer to the list of references at the end.

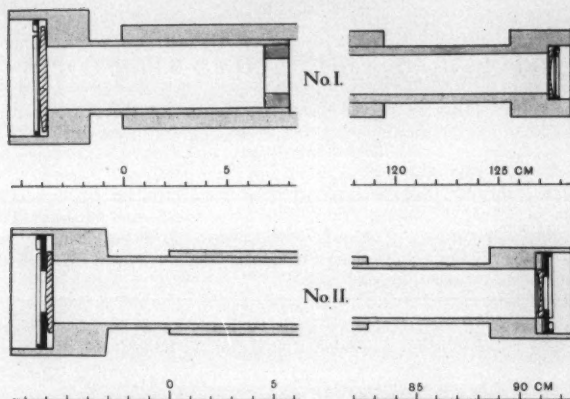


Fig. 2. Replicas of the two Galileo telescopes. Sectional drawings showing internal arrangement.

The particulars which follow are taken from the interesting and valuable papers of Prof. Abetti and Prof. Ronchi.

	F	R_1	R_2	n	D	d	t
O.G. I	132.7	99.55	346.5	1.580	5.1	2.6	0.25
Oc. I	-9.52	∞	4.85	1.500	2.6	1.1	0.30
O.G. II	95.6	53.5	505.0	1.550	3.7	1.6	0.20
Oc. II	-4.88	5.15	5.15	1.527	$\begin{Bmatrix} 2.2 \\ 1.7 \end{Bmatrix}$	1.6	0.18
O.G. III	168.9	94.16	1436.3	1.523	5.8	3.8	0.40

No. III is the broken object-glass. All dimensions are in cm. F = focal length, R_1 and R_2 = radii of curvature, n = refractive index, D = diameter of lens, d = diameter of diaphragm, and t = thickness at centre of lens. The values given for F refer to the central zone, with light of wave-length 5500 A.U. Values of F for other wave-lengths are given below.

λ (A.U.)	O.G. I	O.G. II	O.G. III
4500	130.4	93.8	166.3
5000	131.7	95.0	167.8
5500	132.7	95.6	168.9
6000	133.3	96.2	169.7
6500	133.7	96.5	170.2
7000	133.9	96.9	170.5

The peripheral zone, without stop, in O.G. I has a focal length of 130 cm. (with $\lambda = 5500$), or 2.7 cm. shorter than that for the central zone. In O.G. II the corresponding difference is 2.2 cm.

The O.G.'s of I and II and the ocular of I are of a greenish colour, and their edges are chipped. O.G. III is of a violet colour and has a ground edge; the glass is much less transparent than the ocular of II, which is white with a slight tinge of red. The ocular of II is undoubtedly of later date than the other lenses; in shape, size, and workmanship it resembles the Divini oculars preserved at Florence.

A wooden telescope tube was made by Prof. Ronchi to take O.G. III and Oc. I, and this combination was used by Prof. Abetti in his test observations. The magnifications, etc., of these telescopes are approximately as follows:

Telescope	Magnification	Field of view, mins. of arc	Resolving power secs. of arc
I	14	15	15
II	20	15	10
III	18	not given	10

O.G. II is smaller than and inferior to O.G. I, and the relatively better performance of telescope II is due to the better ocular.

In his paper Prof. Ronchi gives photographs showing the results of his tests, employing his own methods (4), with a rectilinear grating having 10 lines per mm., Foucault's knife-edge method, and the monochromatic artificial star method. These show clearly that, while O.G.'s I and II are bad, O.G. III represents a real superiority.

The following observations are recorded in Prof. Abetti's paper:

Telescope I. The photosphere of the sun, devoid of spots, shows the characteristic granular structure, and the edge sufficiently well defined, with little colour. On the 24th of May, two spot nuclei, about 17 seconds of arc apart, appear fairly well separated.

The moon, during the first quarter, gives a luminous image, but generally not well defined. Copernicus at the edge can be seen well, also Eratosthenes. One has the impression that more details are visible than are shown in the original drawings of Galileo, which are preserved among his manuscripts in the National Library in Florence.

Jupiter gives an image not well defined and which appears double for certain positions of the eye. On the 23rd of May, the more distant satellites I and IV are easily seen; the other two, separated by 20 seconds of arc at the time of the observation, are seen separated by Hale. On the 24th of May, satellites II, III and IV are well visible; satellite I at the time of the observation is approaching Jupiter's disc, is about 15 seconds of arc away, and is seen with difficulty.

The companion of Mizar, which is about 14 seconds of arc from the principal component, is discernible with difficulty, owing to the distortion of the images. Saturn, whose ring is not very open, the minor axis being only one-seventh of the major axis, is seen as an elongated disc, without any detail. It is just the appearance described by Galileo himself in a letter to Julian de Medici on the 13th of November 1610, in which, narrating that the planet is formed of three stars which nearly touch, he adds, however, that if they are viewed with a telescope of not very great magnification, they will not appear three distinct stars, but a longish star of the shape of an olive, thus $\circ$.

Telescope II. The images obtained with this telescope are less luminous than those for No. I, and the magnification, which is greater, causes more detail to be visible, but in reality the images are less clear than those given by No. I. In the moon, the details of minimum diameter visible (such as the crater of Lambert) measure from 15 to 20 seconds of arc. Jupiter and Saturn appear somewhat as with the previous telescope, Mizar being better separated; the limit of separation for this telescope can be considered to approach 10 seconds of arc. On the sun, the photosphere and spots are as with No. I.

Telescope III. Notwithstanding that the O.G. is broken in several pieces, it is easily seen that it is better than the other two O.G.'s. On the moon are visible more details, e.g. craters of the dimensions of Herschel (10 to 15 seconds of arc).

Jupiter and Saturn appear somewhat as with the others, but the satellites are more easily and distinctly visible. On the 30th of June, a small group of spots with 5 nuclei appearing on the sun is easily visible; the distance between the nearest nuclei is 15 seconds of arc. It can be concluded that the resolving power of this combination is about 10 seconds of arc.

On the question of authenticity, as investigated by Favaro⁽¹⁾, Ronchi, Abetti, and others, there is practically no doubt that O.G. III was made by Galileo; with regard to I and II there is some doubt, but the great probability is that they were made by Galileo, and that they were amongst the first made by him. He certainly did make better ones.

I have to thank Col. H. G. Lyons for allowing me to use the photographs in Fig. 1, and Prof. Abetti and Prof. Ronchi for providing information referring to these and other early telescopes.

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REVIEW

Penrose's Annual: The Process Year Book. Vol. 26. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S. Pp. xvi + 136. (London: Percy Lund, Humphries and Co., Ltd., 1924.) Price 8s. net.

The present volume contains the following articles: "Review of Process Work," "On Orthochromatizing and Panchromatizing Gelatino-Bromide Paper," "A Right Spirit in Publishing," "Tri-colour Engraving, Offset and their New Competitor," "The Ashendene Press, 1895-1923," "On Printing without Overlays," "A Renaissance of Wood Engraving," "The Poster and Symbolism," "Handicraft in Bookbinding," "Modern Swedish Book Production," "A Revolution in Lithography," "Co-operation," "Publishers' Book Jackets," "Photoline," "Whenever I see a 'Fine' Book," "The Synchronizing of Offset with Photo-Lithography," "A Universal Composing Machine," "Some Practical Hints on Rotary Photogravure," "Etching on Celluloid," "By paths of Photography," "A Tried and Proven Engraving Machine," "A School for Critics," "What Process Work owes to New York," "Step and Repeat," "Some Important Points in Screen Negative Making," "A Grained Screen," "Metallic Inks," "The Bawtree Photographic Type-Composing Machine," "The Colorway," "Photo-Litho Formulae," "Ideals in Half-Tone Emulsion," "The Cold Top Enamel Process," "Process Screens," "Highly Sensitive Bitumen," and "A Note on the Baskerville Type." The book contains numerous illustrations representing process work during the year 1923, in addition to plates illustrating the articles.

J. S. A.

ABSTRACTS OF PATENT SPECIFICATIONS

206208. Lambert, J. A., and another.

LIGHT-FILTERS.

In order to produce a daylight effect, the light from a gas-filled metal-filament lamp is passed through a moving filter consisting of dark blue, light blue, and clear portions.

206784. Thornton, A. A.

ADJUSTING LENSES.

In a certain kinematograph projector system it is necessary to adjust a number of lenses upon a disc, so that their optical centres are equally spaced around a circle. This adjustment is effected by the use of an eyepiece which, combined with each of the lenses in turn, forms a telescopic system. The adjustment is correct when the replacement of one objective by another, or the rotation of the whole system about the axis of the eyepiece, produces no apparent movement of the target.

207184*. Goerz Optische Anstalt Akt-Ges.

RANGE-FINDERS.

A telescope is arranged in conjunction with sighting marks, adjusting devices, and auxiliary mechanical apparatus, to determine the position which will be taken up after a certain interval of time by an object moving uniformly in a horizontal line, at a height assumed known. The target is first followed for a definite period to enable the necessary settings to be made.

* Specification published before acceptance.

207454. McConkey, J.

SEXTANTS.

A graduated bar carries the zenith pin, and also a member the upper edge of which forms an artificial horizon, which can be brought into coincidence with the image of the pin in the horizon mirror. An extended or additional horizon is formed by the returned end of an adjustable arm secured by clamping nuts. A special mounting is provided for the index mirror.

207539*. Stutzer, R.

DIRECTION-FINDERS.

An open or telescopic sight moving with a gun is directed upon a condenser having a wide-angle field occupied by a graduated scale. A disturbance of the gun, as by the recoil, can thus be determined in amount, and rectified.

207842. MacLeod, M. N.

PHOTO-SURVEYING.

In the production of maps from aerial photographs, the height and tilt of the camera are arrived at by mounting the photograph on an adjustable carrier, and viewing it through a screen upon which the position of known points have been plotted.

207871. Watts and Son, Ltd., and another.

LEVELS.

In order that the length of the bubble may remain sensibly constant, the volume of the tube is reduced by flattening it in cross section, or inserting a glass rod. The bubble itself is of relatively large size.

208128*. Goerz Optische Anstalt Akt-Ges.

RANGE-FINDERS.

The apparatus is designed to predetermine the azimuth and angle of sight of a target moving uniformly in a horizontal line, at a height assumed known. Points corresponding to successive observed positions of the targets are plotted to scale upon a diagram by mechanism coupled to the sighting-tube, and by means of a suitable multiplying gear a pointer is moved to show the position which will be taken up after a predetermined time.

208139. Zeiss, C.

LENS SYSTEMS.

A Galilean telescopic system consists of two positive cemented doublets, with the glass of lower refractive index facing the incident light, and a cemented triplet eyepiece, with an inner flint component.

208266. Barr and Stroud, Ltd., and others.

RANGE-FINDERS.

The direction of deviation of an enemy ship from the broadside-on position is determined by the use of two instruments capable of measuring small angles, mounted widely apart on the observer ship. The angle subtended by the bow and stern of the target is measured by each instrument, and corrected by electrical or mechanical means for the difference in distance of the object from the two stations in order that a proper comparison may be made.

208388. Hughes, G. A.

PROJECTORS.

A searchlight is provided with a main aperture for projecting a normal parallel beam, and a supplementary aperture, through which a divergent beam may be thrown, preferably by tilting the reflector.

208633. Fennell, H.

BIFOCAL LENSES.

Into a recess in the main lens is fused a composite disc consisting of a flint segment and a cover glass of the same refractive index as the main lens.

* Specification published before acceptance.

208798. Marten, T. H.

OPTICAL PROJECTORS.

Two projector heads are brought into action alternately by a rotating shutter with reflecting blades. The system may be used in connection with a kinematograph projector to surround the picture with a secondary image, so as to produce the effect of depth.

209038*. Soc. d'Optique, &c. de Haute Précision.

RANGE-FINDERS.

The displacement of the member used for determining the range is converted into a movement of a second member which is inversely proportional to that of the first. A fixed frame carries a guide in the form of a hyperbola, and a sliding carriage carries a roller bearing upon the guide. The transverse and longitudinal movements of the roller are transmitted to pinions which are rotated by amounts which are inversely proportional to each other.

209041*. Établissements E. Belin.

OPTICAL SYSTEMS.

An optical system for producing a number of images of an object consists of a large objective which renders the light from the object parallel, and a number of small objectives, each placed in front of an aperture in a diaphragm situated between them and the large objective.

209042*. Del Valle, J. M. G.

FOCUSING DEVICES.

A rod equal in length to the focal length of the lens is fixed to the lens carrier at right angles to the axis of the bench, and to it are pivoted two rods, which are fixed at right angles to each other. These two rods pass through guides, which are pivoted to carriages sliding upon the bench, and carrying at distances from the pivots equal to the focal length the object and image surfaces respectively. By this means the focus is automatically maintained.

209093*. Goerz Optische Anstalt Akt-Ges.

LENSES.

Relates to triplets comprising (a) a positive meniscus next to the diaphragm, (b) a bi-concave central element, and (c) a bi-convex outer element. The bi-concave lens (b) is made of a flint glass having a refractive index at least $\cdot 1$ higher than that of the meniscus (a), and the bi-convex lens (c) is made of a baryta flint having a higher refractive index and dispersion than the glass of the lens (b).

209255. Établissements E. Belin and another.

OPTICAL INDICATING SYSTEMS.

Light passing through a slotted diaphragm is focussed by a condenser and a short focus lens, so that an image of the slit is formed upon a fine fibre. In the normal position the light is wholly obstructed, but a slight movement of the fibre allows light to pass. Weak telegraphic signals, for instance, may thus be observed, or recorded photographically.

209371. Lee, H. W., and another.

LENSES.

Improved zonal spherical correction and apochromatism are claimed for a system consisting of a simple collective component, a simple dispersive component, and a more widely spaced compound collective component. This third component has a collective cemented surface, with the concavity turned inwardly, and a radius of curvature not greater than one half of the focal length of the complete system.

209609. Taylor, A., and another.

REFLECTORS.

The bubble of a spirit level is observed in a mirror having two portions arranged at a small angle to each other, so that the ends of the bubble appear to be brought close together.

* Specification published before acceptance.

210003. Barr and Stroud, Ltd., and others.

RANGE FINDERS.

In apparatus as described in specification No. 208266 for determining an enemy's course, the necessity for making simultaneous observations from the two stations is obviated by arranging that the observations at the subsidiary station are transmitted to the main station, and the non-simultaneous observations recorded upon a travelling band. The continuity or separation of the two series of marked points gives the required indication.

210049*. Caméléon Soc. Anon.

OPTICAL PROJECTION.

A number of positives upon a single base are differently coloured, so that they may be projected separately by using light of the appropriate colour.

210352. Hart, E.

GRINDING AND POLISHING GLASS.

Abrasive and liquid is fed to the work from a tank through a constricted opening which is normally clogged by the abrasive. Compressed air is periodically admitted to the feed pipe, and has the effect of clearing the opening, and at the same time mixing up the abrasive and liquid by bubbling through the container.

210386*. N.-V. Machinerieën en Apparaten Fabrieken.

AERODROME LIGHTS.

The source of light is placed a little above the axis of a dioptric lens, thus throwing a beam of light downwardly to mark out the landing space. A second beam, interrupted intermittently by a shutter, is thrown upwards through the top of the lamp to form a beacon light.

210411. Hammond, L.

STEREOSCOPIC SHADOWGRAPHS.

Two shadows of an object, formed by different sources of light, are so arranged as to be seen respectively by one eye only. The two sources may be of complementary colours, corresponding spectacles being employed in viewing, or observation may be made through shutters, run in synchronism with shutters in the projection system. By suitably positioning the sources of light, the shadows may be made to appear to be floating in space.

210533. Reid, R. C.

GRINDING AND POLISHING LENSES.

Lenses are ground and polished upon a reciprocating and rotating wheel, shaped in axial section to the curvature to be imparted. The lenses are pressed against the wheel, and are mounted on carriers which rock on centres so as always to remain in contact with it.

210705. Taylor, A., and another.

LENSES.

A projector consisting of three separated simple components, a negative between two positives, is designed to work with a diaphragm placed well in front of the first principal point. It is adapted to be used with cinematograph projectors in which a continuously moving film is employed, in conjunction with a ring of mirrors.

210763*. Zeiss, C.

SURVEYING INSTRUMENTS.

In tacheometers in which two images are produced by the inclusion in the system of a pair of adjustable half-lenses, one at least of the half-lenses is automatically displaced, as the elevation of the telescope is varied, so that the readings are reduced to the horizontal plane.

* Specification published before acceptance.